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COMPUTING PLANETARY POSITIONS: USER-FRIENDLINESS AND THE ALFONSINE CORPUS

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Astronomical tables are ways to turn the treatment of complex problems into elementary arithmetic. Since Antiquity astronomers have addressed many problems by means of tables; among them stands out the treatment of planetary motion as well as that for the motions of the Sun and the Moon. It was customary to assign to the planets constant mean velocities to compute their mean longitudes at any given time in the past or the future, and to add to these mean longitudes corrections, called equations, to determine their true longitudes. In this paper we restrict our attention to the five planets,[1] with an emphasis on their equations. Section 1 deals with what we call the standard tradition, beginning with Ptolemy's *Handy tables*, and Section 2 deals with the new presentations that proliferated in Latin Europe in the fourteenth and fifteenth centuries, some of which reflect a high level of competence in mathematical astronomy.[2]

1. *The Standard Tradition*

By the middle of the second century A.D. Ptolemy displayed tables for the equations of the five planets with specific layouts and based on specific models, algorithms, and parameters. We argue that this category of tables, as is the case for many others, provides a clear example of user-friendliness, the driving force that prevailed in the history of table-making.

In *Almagest* XI.11 Ptolemy presented tables for the planetary equations, one for each of the five planets.[3] Each table has eight columns, of which the first two are for the argument (one from 6º to 180º and the other for its complement in 360º). The argument is given at intervals of 6º, from 6º to 90º (and for 270º to 354º), and at intervals of 3º, from 90º to 180º (and for 180º to 270º). According to Toomer, Ptolemy computed the entries at 6º-intervals, even where the function is tabulated at 3º-intervals.[4] Columns 3 and 4 are for the equation in longitude and the difference in equation, respectively. Column 3 assumes an eccentric model, which Ptolemy rejected in favour of an equant model. Column 4 displays the difference between the equation for an equant model and the equation for an eccentric model. The sum of corresponding entries in these two columns is the equation of centre, which replaced columns 3 and 4 that appear in *Almagest* XI.11 (see Table A, col. 3).[5] Columns 5 and 7 give the subtractive and additive differences to be applied to the equation of anomaly (displayed in col. 6), when the planet is at greatest and least distance, respectively. Column 8 is for the minutes of proportion, to seconds, used for interpolation purposes. We note that, in the case of Venus, the entries for the equation in longitude (col. 3) are exactly the

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same as those for the solar equation, although Ptolemy does not call attention to this fact.[6] We display Ptolemy's model for Mars to illustrate how a planet's position can be computed directly from the model: see Figure 1. To do this, one must solve plane triangles by means of trigonometric procedures that were already available in Ptolemy's time. The solution is as follows. Given $\bar{\kappa}$, we wish to compute the correction angle, c_3, by solving triangle ECO. But, before we can do this, we have to find the length of EC, where DC, the radius of the deferent, is 60. So first we must solve triangle EDC to find EC, where angle CED is the supplement to angle $\bar{\kappa}$ and ED is the eccentricity (a given parameter in the model). With $\bar{\kappa}$, EC and EO (twice the eccentricity), we can solve triangle ECO, which yields the values for c_3 and CO. We then have to solve triangle MCO to find $c(\alpha)$. In this triangle two sides and an angle are known: angle MCO is equal to $180° - (\bar{\alpha} - c_3)$, CM is the radius of the epicycle (a given parameter in the model), and CO has already been determined. Then

$$\lambda = \lambda(A) + \bar{\kappa} + c_3 + c(\alpha),$$

where $\lambda(A)$, the longitude of the apogee, is a given parameter in the model. Using the planetary equation tables takes trigonometric functions out of the computational scheme.

In the *Handy tables* Ptolemy did not modify the models or the parameters for the planetary equations, but he introduced a series of changes to make the tables more suitable for calculation. Firstly, the arguments are now given at intervals of 1°, rather than at intervals of 3° or 6°, as was the case in the *Almagest*.[7] This certainly simplifies interpolation. Secondly, he merged columns 3 and 4 in the *Almagest* into a single column representing the equation of centre, thus reducing the number of operations required for using these tables. This also reduced the number of columns, from 8 to 7. Thirdly, the column for the minutes of proportion was also modified by avoiding unnecessary precision (the entries are given to seconds in the *Almagest* but only to minutes in the *Handy tables*) and by changing the argument (mean argument of centre in the *Almagest* and true argument of centre in the *Handy tables*).[8] This new presentation (see Table A for Mars) set the standard for most tables dealing with planetary equations for about fourteen centuries.

In Table A, the mean argument of centre, $\bar{\kappa}$, serves as argument (columns 1 and 2) for the equation of centre (col. 3), c_3; the true argument of centre, κ, can then be computed, for

$$\kappa = \bar{\kappa} + c_3(\bar{\kappa}),$$

where $c_3(\bar{\kappa}) \leq 0°$ when $0° \leq \bar{\kappa} \leq 180°$. The mean argument of centre serves also as argument for the minutes of proportion (col. 4), which are necessary to compute the true position of the planet when not found at maximum or minimum distance of the epicycle from the observer. Now the true argument of anomaly, α, serves as argument (cols 1 and 2) for the equation of anomaly (col. 6), and is obtained from the mean argument of anomaly:

$$\alpha = \bar{\alpha} - c_3(\bar{\kappa}).$$

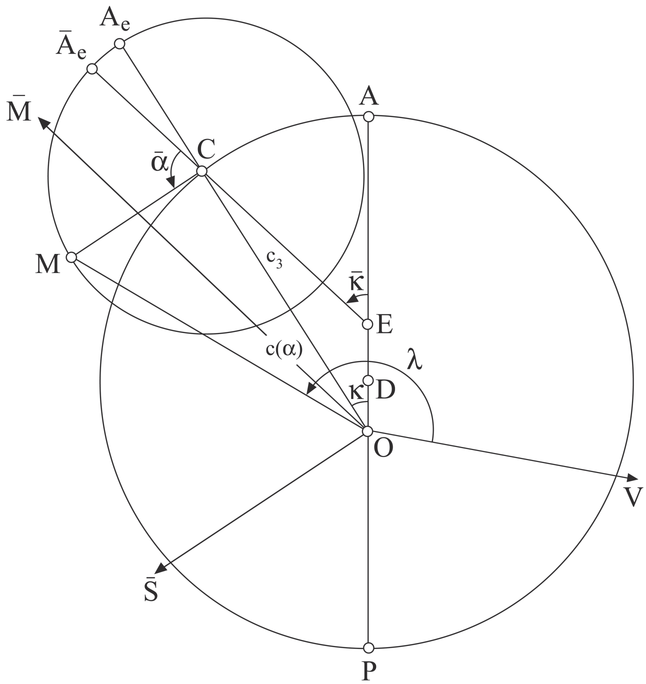

FIG. 1. Ptolemy's model for Mars. The observer is at O; the centre of the deferent, AC, is at D; the equant is at E such that ED = DO; the direction to the vernal equinox is OV; A is the apogee, C is the centre of the epicycle such that DC = 60. The radius of the epicycle is CM; the mean apogee of the epicycle is at $\bar{A}_e$ and the true apogee of the epicycle is at A_e; the mean longitude of Mars is in the direction $O\bar{M}$, and its true position is at M. The mean argument of centre is $\bar{\kappa}$ and the mean argument of anomaly is $\bar{\alpha}$; the equation of centre is c_3, and the equation of anomaly is $c(\alpha)$. The true longitude of Mars, λ, is angle VOM. For an outer planet the direction CM is always parallel to $O\bar{S}$, where $\bar{S}$ is the direction to the mean Sun.

The tabulated equation of anomaly, $c_6(\alpha)$, was originally computed assuming that the centre of the epicycle is at mean distance. When the epicycle lies between maximum distance (apogee) and mean distance, a subtractive difference (col. 5) must be applied. Similarly, when the epicycle is between minimum distance (perigee) and mean distance, an additive difference (col. 7) must be applied. The true argument of anomaly serves as argument for both subtractive and additive differences. Then, the total equation of anomaly is

$$c(\alpha, \bar{\kappa}) = c_6(\alpha) - c_5(\alpha) \cdot c_4(\bar{\kappa}),$$

when $\bar{\kappa}$ ranges from 270º to 90º, that is, when the planet is near apogee, and

$$c(\alpha, \bar{\kappa}) = c_6(\alpha) + c_7(\alpha) \cdot c_4(\bar{\kappa}),$$

when $\bar{\kappa}$ ranges from 90º to 270º, that is, when the planet is near perigee. The combined effect of the equation of centre and the total equation of anomaly is

TABLE A. Equations for Mars in the *Handy tables* (excerpt).[9]

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Argument		Equation of centre	Min. prop.	Subtractive difference	Equation of anom.	Additive difference
(°)	(°)	(°)	(′)	(°)	(°)	(°)
1	359	0;11	60	0; 2	0;24	0; 2
2	358	0;22	60	0; 3	0;48	0; 3
3	357	0;33	60	0; 4	1;12	0; 4
…						
86	274		2			
87	273		1			
88	272		1			
89	271		2			
90	270			2;28	33;22	2;49
…						
92	268	11;24				
93	267	11;25				
…						
96	264	11;25				
97	263	11;24				
…						
130	230				41; 9	
131	229				41;10	
132	228				41; 9	
…						
152	208			5;37		
153	207			5;38		
…						
156	204			5;38		
157	203			5;37		
158	202					8; 2
159	201					8; 3
160	200					8; 2
…						
178	182	0;27	60	0;51	3;52	1;35
179	181	0;14	60	0;26	1;57	0;48
180	180	0; 0	60	0; 0	0; 0	0; 0

thus $c_3(\bar{\kappa}) + c(\alpha, \bar{\kappa})$, and the true position of the planet, λ, at a given time is:

$$\lambda = \bar{\lambda} + \bar{\kappa} + c_3(\bar{\kappa}) + c(\alpha, \bar{\kappa}),$$

where $\bar{\lambda}$, the mean longitude of that planet at a given time t since epoch, is defined as:

$$\bar{\lambda} = \lambda_0 + \Delta\lambda \cdot t,$$

λ_0 being the planet's mean longitude at epoch, and $\Delta\lambda$ the planet's mean motion in longitude.

In the early Islamic world, the *Zīj al-Sindhind* of al-Khwārizmī (*fl.* 830) followed the Indo-Iranian tradition, which was not based on Ptolemaic models and parameters, and made no use of equants.[10] This tradition was represented by the *Zīj al-Shāh*, a work composed in Sasanian Persia and translated into Arabic *c.* 790, where the maximum value for the equation of Venus is set equal to that of the solar equation (2;13° or 2;14°); the identity of these parameters is also found in the *Almagest*.[11] Accordingly, the tables for the planetary equations are quite different, both with respect to the entries and the presentation, from those in the *Almagest* or the *Handy tables*.

The Greek tradition was represented in the eastern Islamic world by the *Zīj al-Ṣābi'* of al-Battānī (d. 929) which is strongly Ptolemaic; indeed, the tables in it for the planetary equations followed exactly those in the *Handy tables*, but for the equation of centre of Venus.[12] Both the *Almagest* and the *Handy tables* have 2;24º as the maximum value for Venus's equation of centre, whereas it is 1;59º in the zij of al-Battānī. This change in the equation of centre of Venus was not due to new observations of Venus; rather, it was the result of a new value found from observations for the eccentricity of the solar model that implied a maximum solar equation of 1;59,10º. This new solar parameter was simply applied to the equation of centre for Venus, where it was rounded to 1;59º. In this modification al-Battānī followed other Islamic zijes, such as that of Ḥabash al-Ḥāsib (*fl.* 850).[13] Toomer pointed out that modifying the entries for the equation of Venus was inconsistent with leaving unchanged the entries for the subtractive and additive differences (at greatest and least distances, respectively), because they also depend on eccentricity.[14] In any case, in the zij of al-Battānī only the entries for the equation of centre of Venus differ from those in the *Handy tables* whereas all the rest remain unchanged.

The Toledan Tables were compiled in the second half of the eleventh century, but the original Arabic version is not extant. In the Latin versions of the Toledan Tables the presentation and the numerical entries agree with those in the zij of al-Battānī, but for (in most cases) an added column for the first station of each of the planets.[15] In *Almagest* XII.8 Ptolemy displayed the first and second stations of the five planets in a single table, using the mean centre as argument, with entries at intervals of 6º.[16] In the *Handy tables*, Ptolemy gave more entries, at 3º-intervals, and presented a table for the two stations for each planet. He also introduced a change in the argument (true argument of centre, instead of mean argument of centre), thus making the entries slightly different from those in the *Almagest*.[17] In his zij al-Battānī reproduced in a separate table the entries for the first and second stations in the *Handy tables*, and only displayed them at 6º-intervals. The compilers of the Toledan Tables probably realized that it was unnecessary to give entries for both the first and the second stations (because corresponding entries add up to 360º) and just included a specific column for the first station of each of the planets. Thus, in the tables for the planetary equations, ultimately derived from the *Handy tables*, the number of entries increased, for they are given here at intervals of one degree, and gained one column which was eliminated as a separate table.[18] The Toledan Tables were by far the most popular tables in Latin Europe, and the presentation in them for tables of planetary equations can be considered standard.

The Maghribī astronomers Ibn Isḥāq al-Tūnisī (*c.* 1193–1222), Ibn al-Bannā' of Marrakesh (1265–1321), and Ibn al-Raqqām (Tunis and Granada, d. 1315) used new parameters for the equations of centre of Saturn, Jupiter, and Venus. In contrast, the values given to the equations of anomaly agreed precisely with those in the standard tradition, namely that of the *Handy tables*, the zij of al-Battānī, and the Toledan

Tables, but for the fact that in this tradition the columns displayed are combinations of cols 5, 6, and 7.[19] We further note that the tables for the planetary equations of Ibn Isḥāq and his followers depart from the standard tradition not only in the three basic parameters already mentioned, but also in presentation. Indeed, for each of the planets there are two tables of equations: one for quantities that depend on the argument of centre and one for those that depend on the argument of anomaly.[20]

The Castilian Alfonsine Tables were produced in Castile by two astronomers working under the patronage of Alfonso X (reigned: 1252–84), Judah ben Moses ha-Cohen and Isaac ben Sid. We do not know how the tables for the planetary equations were presented in these tables, because the tables themselves are not extant. However, the canons have been preserved in Castilian, and chapter 18 (*De la equaçion de los V planetas*) describes the way to compute planetary longitudes by means of tables. Although no numerical values are given, the description agrees perfectly with the layout of tables in the standard tradition of the *Handy tables*, the zij of al-Battānī, and the Toledan Tables.[21]

This tradition was transmitted from the Iberian Peninsula to the rest of Europe. The earliest astronomer to depend on this Iberian tradition was Jean Vimond, who was active in Paris *c.* 1320. He compiled a set of tables that appear to be at the interface of the astronomy rooted in al-Andalus and the Maghrib and developed in Castile in the late thirteenth century on the one hand, and the activity of the astronomers working in Paris in the 1320s and the 1330s that resulted in the Parisian Alfonsine Tables on the other.[22] In many ways Vimond's tables follow a tradition unattested in Latin prior to 1320; for example, his tables for the planetary equations are also split into two tables for each planet, much as the Maghribī-Andalusian astronomers did.

In addition to changes in structure, which will be examined later, the main modification in Vimond's tables is found in the entries for the equations of centre of Jupiter and Venus, with maximum values of 5;57° and 2;10°, respectively. Not much can be said about the value 5;57° other than it does not appear in any text or table prior to Vimond's tables, and no medieval discussion of its origin has been found. However, the value 2;10°, also used by Vimond as the maximum solar equation, appears in previous texts: implicitly in a table for the daily solar positions for 1278 contained in the *Libro del astrolabio llano* composed by the astronomers in the service of King Alfonso X of Castile,[23] and explicitly in an account in John of Murs's *Expositio* of two observations of autumnal equinox, one by Ptolemy in 132 and the other attributed to Alfonso in 1252, where John explains that he has seen this observational report in what he calls the "Tables of Alfonso".[24] We thus think it likely that these two new values for Jupiter and Venus/Sun were taken from an earlier work, and the most reasonable candidate is the Alfonsine Tables in the original Castilian version.[25]

The Parisian Alfonsine Tables, produced in the 1320s by a group of astronomers working in Paris, were built on material coming from the Iberian Peninsula. They are best known today from the *editio princeps* that appeared in Venice in 1483. While each part of this printed edition has a complicated history, the planetary equation tables in it are faithful to the Parisian Alfonsine Tables as they were presented in the

1320s. The layout of the tables for the planetary equations conforms to the standard tradition, although they have no additional column for the first station. We will refer to the presentation and parameters of this version of the Alfonsine Tables as "standard". The entries are given at intervals of one degree, as was already established in the *Handy tables*.[26] Moreover, out of ten basic parameters for the five planets, only two differ from those defined by Ptolemy, namely, the equations of centre of Jupiter and Venus, and both of these parameters are already found in John Vimond's tables. It is difficult to find other examples of such great stability in the transmission of astronomical tables for more than thirteen centuries. Table B provides a summary of the main parameters for the equations of centre and anomaly used by different authors.

TABLE B. Maximum values of the equations of centre and anomaly in various sets of tables (new values are shown in boldface type).

	Saturn		Jupiter		Mars		Venus		Mercury	
	Eq. of centre	Eq. of anom.	Eq. of centre	Eq. of anom.	Eq. of centre	Eq. of anom.	Eq. of centre	Eq. of anom.	Eq. of centre	Eq. of anom.
Almagest *	6;31°	6;13°	5;15°	11; 3°	11;25°	41; 9°	2;24°	45;59°	3; 2°	22; 2°
Handy tables	6;31°	6;13°	5;15°	11; 3°	11;25°	41;10°	2;24°	45;59°	3; 2°	22; 2°
al-Khwārizmī	**8;36°**	**5;44°**	**5; 6°**	**10;52°**	**11;13°**	**40;31°**	**2;14°**	**47;11°**	**4; 2°**	**21;30°**
al-Battānī	6;31°	6;13°	5;15°	11; 3°	11;25°	41; 9°	**1;59°**	45;59°	3; 2°	22; 2°
Toledan Tab.	6;31°	6;13°	5;15°	11; 3°	11;24°	41; 9°	1;59°	45;59°	3; 2°	22; 2°
Maghribī astr.	**5;48°**	6;13°	**5;41°**	11; 3°	11;25°	41; 9°	**1;51°**	45;59°	3; 2°	22; 2°
Vimond **	6;31°	6;13°	**5;57°**	11; 3°	11;24°	41; 9°	**2;10°**	45;59°	3; 2°	22; 2°
Parisian Alf.	6;31°	6;13°	5;57°	11; 3°	11;24°	41;10°	2;10°	45;59°	3; 2°	22; 2°

* The values for the equation of centre shown here are found by adding algebraically the equation in longitude and the difference in equation in *Alm*. XI.11 (cols 3 and 4).

** The values for the equation of centre shown here result from subtracting the *motus completus* (col. 2) from the mean argument of centre (col. 1). The values for the equation of anomaly shown here result from adding the *motus completus* (col. 2) to the correction for maximum distance (col. 5 in the standard tradition); see below.

2. *A Proliferation of New Presentations*

Prior to the first edition of the Parisian Alfonsine Tables in 1483, a variety of original approaches for presenting tables for the planetary equations were undertaken within the Alfonsine corpus. They coexisted with the standard tradition, which is preserved in a number of manuscripts dating from the fourteenth and fifteenth centuries.[27] The goal of these new approaches that depart from the standard tradition was, once again, to facilitate computation.

Let us return to about 1320, the date of John Vimond's tables, in which the two equations for each planet are displayed in different tables. In those where all the tabulated functions depend on the mean argument of centre (see Table C), the entries are given at 6°-intervals. Vimond displayed the true argument of centre (col. 2: *motus completus*) and added columns for the increment of the true argument per degree of the argument (col. 3: *motus gradus*), planetary velocity (col. 4: *motus diei*), minutes of proportion (col. 5: *diametri*), and first station (col. 6). Moreover, the equation of centre incorporates a displacement which is the difference between the apogee of each of the planets and that of the Sun (no displacement is therefore needed in the case of Venus, for its apogee is assumed to be the same as that for the Sun). Analysis of Vimond's tables shows that the motion of the solar apogee was included in the motions of the planetary apogees, thus following a theory for which there was no previous evidence outside al-Andalus and the Maghrib.[28] With this particular arrangement Vimond intended to present a more user-friendly table than the standard table for the equation of centre.

TABLE C. John Vimond's equation of centre and first station of Mars (excerpt).

(1) Argument		(2) *Motus completus*		(3) *Motus gradus*	(4) *Motus diei*	(5) *Diametri*	(6) First station	
s	(°)	s	(°)	min	min	min	s	(°)
0	6	0	12;31	50;50	26	6	5	8;41
0	12	0	17;36	50; 0	26	4	5	8;21
…								
1	12	1	12;22	49; 0	26	0	5	7;29
1	18	1	17;16	49;10	25	0	5	7;31
…								
4	18	4	6;36	60;30	31	32	5	13;46
…								
7	12	7	11;33	73;30	38	60	5	19;14
7	18	7	18;54	73;10	38	59	5	19;13
…								
10	12	10	23;24	58;50	30	31	5	13;36
…								
11	24	12	2;13	51;50	27	10	5	9;31
12	0	12	7;24	51;10	26	8	5	9; 6

Now, in the tables where all the tabulated functions depend on the argument of anomaly (given at 6°-intervals for Saturn, Jupiter, and Mercury, and at 3°-intervals — and at 2°-intervals in the vicinity of 180° — for the other two planets), Vimond also added columns for planetary velocities and other corrections, such as col. 5 (see Table D), which results from adding the correction for maximum distance to the correction for minimum distance (cols 5 and 7, respectively, in the *Handy tables*, the zij of al-Battānī, and the Toledan Tables). As was the case for the equation of centre, the entries for the equation of anomaly are not explicitly displayed. Rather, we are given entries for the *motus completus* (col. 2), which is the difference between the equation of anomaly and the correction for maximum distance. When we compute the differences between cols 6 and 5 in the standard tradition, we find agreement with Vimond's *motus completus*, indicating that he kept all the basic parameters for the equation of anomaly in this tradition. It is noteworthy that, as indicated by North, this implies that Ptolemy's eccentricities underlie these tables even though, in the case of Venus and Jupiter, the eccentricities were modified for computing the equation of centre.[29] The only text of which we are aware that treats the equation of anomaly in this way is of Maghribī origin: the *Minhāj* of Ibn al-Bannā', dependent on the zij of Ibn Isḥāq. In the *Minhāj* the tables for the equations of anomaly of Saturn and Jupiter give entries for *al-mufrad* ($c_6 - c_5$ in the standard terminology for columns) and *al-bu*ᶜ*d* ($c_5 + c_7$).[30] These are precisely two of the columns found in Vimond's tables (cols 2 and 5). This particular choice of columns was intended to facilitate the computation of the planetary equations of anomaly.

TABLE D. John Vimond's equation of anomaly for Mars (excerpt).

(1) Argument		(2) *Motus compl.*	(3) *Motus gradus*	(4) *Motus diei*	(5) *Diametri*	(6) *Motus grad.*	(7) *Motus diei*
s (°)	s (°)	(°)	min	min	min	sec	sec
0 3	11 27	1; 8	23	11	0; 8	3	1
0 6	11 24	2;16	23	11	0;17	3	1
…							
4 3	7 27	36;40	1	1	8;53	9	4
4 6	7 24	36;44	0	0	9;19	9	4
4 9	7 21	36;43	3	1	9;46	9	4
…							
5 6	6 24	28;15	46	21	13;30	0	0
5 9	6 21	25;56	53	25	13;37	6	2
5 12	6 18	23;17	62	29	13;19	13	6
…							
5 28	6 2	3; 1	90	42	2;29	74	35
6 0	6 0	0; 0	90	42	0; 0	74	35

In addition to the *Expositio*, already mentioned, John of Murs, a key figure in the Parisian milieu for the transmission of Alfonsine astronomy, was responsible for a set of tables, called the Tables of 1321, devoted exclusively to the planets and the two luminaries. With these tables the computation of true planetary positions is entirely different from that described in any other text of which we are aware.[31] The most significant feature of the Tables of 1321 is a new organizational principle, which does not require the equations of the planets to be displayed explicitly. To be sure, the mean motions of the planets are here presented in tables for the mean conjunctions of each planet with the Sun (*tabula principalis*), and the corrections to be applied for times between consecutive conjunctions are given in double argument tables (*contratabula*). This particular approach meant that astronomers could avoid the typically cumbersome computations for determining true planetary positions, compared with using tables previously available in Latin. One unusual feature of these double argument tables is that the horizontal argument is the mean argument of centre of the planet (at intervals of 12º) and the vertical argument is the "age of the planet", that is, the time after a mean conjunction with the Sun, expressed as a number of days. It is also noteworthy that for each planet, besides the *tabula* and *contratabula*, we are given a table for its equation of centre and first station. The values of the maximum equation of centre agree in all cases with those used by Vimond, and so do the rest of the entries (given at 6º-intervals in both sets of tables, but presented differently).

The tables of Vimond and those of John of Murs for 1321 certainly made the computation of the true positions of the planets much easier, but their approaches do not seem to have been very popular. The main improvement in that direction came from double argument tables, which greatly simplified computations and only required linear interpolation.[32] John of Lignères (also active in Paris) was probably the first astronomer in Latin Europe to draw up a double argument table combining the equations of centre and anomaly in a single table for each planet in his *Tabule magne* (*c.* 1325).[33] The vertical argument is the mean argument of anomaly (at 6º-intervals in all planets, and also at 3º-intervals from 150º to 180º in the case of Mars and Venus),[34] and the horizontal argument is the mean argument of centre (at 6º-intervals). In Table E we reproduce an excerpt from the table for the combined equation of Venus in John of Lignères's *Tabule magne*, as found in Lisbon, MS Ajuda 52-XII-35, ff. 83r–87v, with the title *Tabula equationum ultimarum veneris*. We note the use of physical signs of 60º, and the inclusion of columns for the differences, to minutes, of successive entries for a fixed value of the argument of anomaly (not displayed here).[35]

TABLE E. John of Lignères's double argument table for the combined equation of Venus (excerpt).

$\bar{\kappa}$ $\bar{\alpha}$ (º)	0,6º (º)	0,12º (º)	...	3,54º (º)	4,0º (º)	4,6º (º)	...	5,54º (º)	0,0º (º)
	m *	m		a	a	a		a	a
0, 0	0; 8	0;16	...	1; 0**	1; 4	1; 8	...	0; 8	0; 0
	a	a							
0, 6	2;22	2;14	...	3;35	3;39	3;42	...	2;38	2;30
0,12	4;50	4;43	...	6; 6	6;10	6;13	...	5; 7	4;58
...									
2, 6	44; 2	43;51	...	47;23	47;24	47;22	...	44;26	44;14
2,12	44;32	44;20	...	48;10	48;10	48; 8	...	44;57	44;44
2,18	44;29	40;18	...	48;33	48;33	48;30	...	44;59	44;43
2,24	43;41	41;44	...	48;13	48;16	48;14	...	44;13	43;57
...									
2,54	13; 9	12;26	...	22;21	16;21	12;21	...	14;32	13;52
2,57	6;23	5;38	...	27;14	19;14	15;14	...	7;44	7; 7
	m	m							
3, 0	0;44	1;28	...	31; 6	24; 7	19; 7	...	0;39	0; 0

* m stands for *minue* (to be subtracted) and a for *adde* (to be added).
** Ms Erfurt: 1;1.

For each planet there is a total of at least 1860 entries (2160 in the case of Mars and Venus) presented as 31 × 60 or 36 × 60 matrices, not taking into account the columns and rows that display the successive differences. None of the entries explicitly corresponds to the maximum values of the equations of centre or anomaly, which could lead to the identification of the tradition to which it belongs (see Table B), but a few entries are easy to track. Let us consider the case when $\bar{\kappa} = 0°$ or 180°. Then $c_3(\bar{\kappa}) = 0°$ and $\bar{\alpha} = \alpha$, and the entries for $\bar{\alpha} = 90°$ reduce to $c_6(90) - c_5(90)$ and $c_6(90) + c_7(90)$, respectively, in the usual terminology for columns. We find agreement in all cases, except for the equation for Mercury at greatest distance (the entry reads 21;32°, whereas computation with the tables in the standard tradition give 22;2°).[36] In all other cases there is good agreement, but it is not always perfect because columns 5, 6, and 7, which depend exclusively on the argument of anomaly, sometimes vary in the minutes. To show that the entries in John of Lignères's table are specifically based on the values used by John Vimond and, in particular, on those maximum values for the equation of centre found for the first time in Vimond's tables, we have recomputed a few critical entries.[37] The maximum entries in John of Lignères's tables could not have been computed with values as low as those in the tradition represented by the Toledan Tables, and we conclude that they were calculated with Vimond's tables, or that both astronomers had a common source.

It should be noted that in his tables of 1322 John of Lignères had used the parameters 1;59° (Venus) and 5;15° (Jupiter) that are found in the Toledan Tables for the maximum equations of centre, replacing them with 2;10° (Venus) and 5;57° (Jupiter) in his double argument tables for the planetary equations in 1325. As a matter of fact, John of Lignères's tables for planetary equations for 1322, as presented in Bibliothèque Nationale de France, MS 7286C (ff. 33r–47v),[38] share the same entries and layout, including a column for first station, with the Toledan Tables. This change was much the same as John of Murs had done a few years previously, given that in his earliest astronomical work of 1317, beginning *Auctores calendarii*…, he had praised the Tables of Toulouse and seemed unaware of Alfonsine material.[39]

Double argument tables undoubtedly facilitated computation, because they displayed in a compact and clever way intermediate calculations needed to obtain a final numerical result.[40] This kind of presentation was not an invention of the Parisian astronomers, for it is already found in Arabic sources, e.g., it was used by Ibn al-Kammād (Córdoba, *c.* 1100) in his tables for the time from mean to true syzygy as a function of the difference between the hourly velocities of the Moon and the Sun and the elongation.[41] Double argument tables proliferated in fourteenth-century Europe and were not restricted to the planetary equations: they were also used to display true planetary positions (the *Tabule anglicane*, also called the Oxford Tables of 1348, associated with William Batecombe); planetary conjunctions (John of Murs's Tables for 1321); planetary latitudes (John of Murs's Tables for 1321, and the Oxford Tables); syzygies (John of Murs and Firmin of Beauval in their *Tabulae permanentes*, Immanuel ben Jacob Bonfils of Tarascon, Levi ben Gerson, Juan Gil of Burgos, Joseph Ibn Waqār of Seville, and the Tables of Barcelona); lunar motion (Levi ben Gerson); and lunar and planetary velocities (Judah ben Asher II of Burgos).[42]

Another set of tables in the Alfonsine corpus that adheres strictly to its parameters and models is the set we call the Tables for the Seven Planets for 1340; they are a most ingenious reworking of the Parisian Alfonsine Tables and include several displaced tables. The purpose of displaced tables is to eliminate all subtractions in the derivation of planetary positions, thus facilitating computations.[43] This anonymous set of tables, most likely of French origin, is uniquely preserved in Paris, Bibliothèque Nationale de France, MS 10262 (ff. 2r–46v). The two planetary equations are given in separate tables for each planet, and are not explicitly displayed. Rather, for the equation of centre we are given entries that are displaced both vertically and horizontally with respect to those in the standard Parisian Alfonsine Tables, whereas the entries for the equation of anomaly are only displaced vertically. In modern algebraic terms, the vertical and horizontal displacements of a function underlying a displaced table are such that $y = f(x + kh) + kv$, where $y = f(x)$ is the original function to which the displaced table is compared, kh is the displacement on the X-axis, and kv is the displacement on the Y-axis. Tables F and G display excerpts of the equation of centre of Jupiter in the Tables for the Seven Planets and in the Parisian Alfonsine Tables, respectively (where kh = 18º and kv = 6º).

Figure 2 illustrates the situation for Jupiter. The graph labelled MS 10262 displays the entries in the Tables for the Seven Planets for 1340, and that labelled PAT corresponds to those in the Parisian Alfonsine Tables.

In general the vertical displacements are intended to avoid complicated rules for addition and subtraction corresponding to the simple rules we now give by means of algebraic signs. The horizontal displacements are intended to counterbalance other displacements, such as those applied to the minutes of proportion. It is easy to recognize that the vertical displacements of the entries for the equation of anomaly agree with the parameters found in the Parisian Alfonsine Tables: see Table B. The Tables for the Seven Planets use a total of 40 different displacements for the planets (including the Sun and the Moon): see Table H.

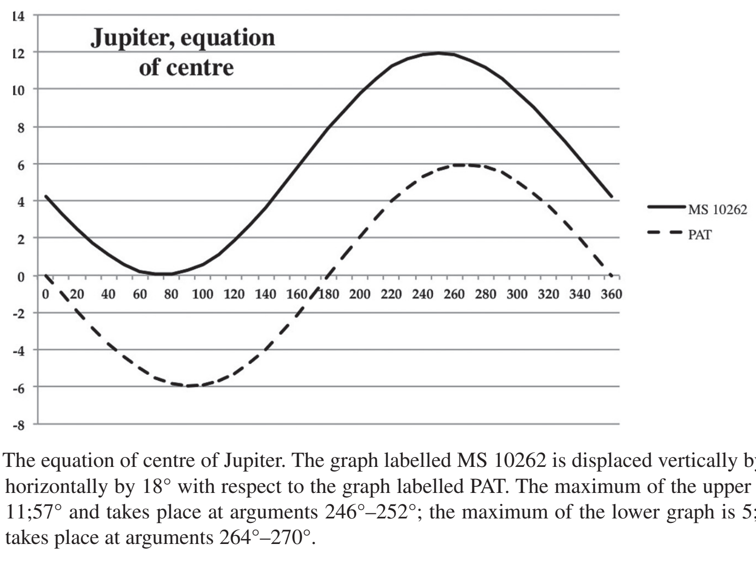

FIG. 2. The equation of centre of Jupiter. The graph labelled MS 10262 is displaced vertically by 6° and horizontally by 18° with respect to the graph labelled PAT. The maximum of the upper graph is 11;57° and takes place at arguments 246°–252°; the maximum of the lower graph is 5;57° and takes place at arguments 264°–270°.

TABLE F. Equation of centre of Jupiter in the Tables for the Seven Planets.[44]

(º)	*Equatio centri* (º)	... Minutes of proportion (′)
0	4;15	0
1	4; 9	1
2	4; 3	1
…		
71	0; 4	53
72	0; 3	54
…		
77	0; 3	59
78	0; 3	2
79	0; 4	3
…		
168	6;39	60
…		
245	11;56	14
246	11;57	13
…		
252	11;57	7
253	11;56	6
…		
259	11;52	1
260	11;51	58
…		
359	4;20	0

TABLE G. Equation of centre of Jupiter in the Parisian Alfonsine Tables.[45]

(º)	Equation of centre (º)	Minutes of proportion (′) ...
1	0; 6	60
2	0;12	60
3	0;12	60
	…	
88	5;56	1
89	5;56	1
90	5;57	2
…		
96	5;57	7
97	5;56	8
…		
180	0; 0	60
…		
263	5;56	8
264	5;57	7
…		
270	5;57	2
271	5;56	1
272	5;56	1
273	5;55	2
…		
358	0;12	60
359	0; 6	60

In any case, computation with this compact and consistent set of tables gives the same results as those obtained with the Parisian Alfonsine Tables, while avoiding subtractions at any stage in the computation.

In the fifteenth century the Paduan astronomer, Prosdocimo de' Beldomandi (d. 1428), compiled a new set of tables that belong to the Alfonsine corpus.[46] His tables for the planetary equations follow the Parisian Alfonsine Tables, including the 2;10º and 5;57º used by Vimond for Venus and Jupiter, in agreement with those that were printed in 1483 in the *editio princeps*.

Giovanni Bianchini (d. after 1469) spent most of his life in Ferrara where he served as administrator for the estate of the prominent d'Este family. About 1442 he compiled an extensive set of astronomical tables that depend on the Alfonsine Tables, but have a completely different presentation.[47] Bianchini's tables offer a whole new approach for computing the true positions of the planets. Although tables for the planetary equations are not explicitly given, the true positions of the planets are computed by means of double argument tables where the vertical argument is the mean anomaly, represented here by the time within an anomalistic period for each planet, and the horizontal argument is the mean centre. These tables were first published in 1495 in Venice under the title *Tabulae astronomiae*, and again in 1526 and 1553.[48]

The *Tabulae resolutae* were compiled in central Europe, and circulated widely

TABLE H. Displacements of the planetary equations in the Tables for the Seven Planets for 1340.

	Eq. of centre		Eq. of anomaly
	Vert. displac.	Horiz. displac.	Vert. displac.
Saturn	7°	14°	6;13°
Jupiter	6°	18°	11; 3°
Mars	12°	61°	41;10°
Venus	3°	51°	45;59°
Mercury	4°	28°	22; 2°

in manuscripts during the fifteenth century and in print during the sixteenth.[49] One of their characteristics is that the mean motions are arranged according a system of cyclical radices at intervals of 20 years. The *Tabulae resolutae* are also strictly based in the Parisian Alfonsine Tables; in fact, they are a particular form of presenting them. The tables for the planetary equations display the same parameters as the Parisian Alfonsine Tables but, unlike them, add a column for first station, thus following the layout of most versions of the Toledan Tables.

In Vienna John of Gmunden (*c*. 1380–1442) collected a great variety of tables within the framework of the Parisian Alfonsine Tables. He displayed them in various sets, called "First Version", *Tabulae maiores*, and *Tabulae breviores*.[50] He presented his tables for the equations of the planets in three different ways: at 1°-intervals following the standard tradition; at 3°-intervals in an abridged form of the latter; and as double argument tables, reproducing those by John of Lignères. Therefore, with respect to the planetary equations, John of Gmunden was not an innovator; rather, he offered table users several possibilities that were already known in Latin Europe.

In the early sixteenth century, Johannes Angelus, a follower of Peurbach and Regiomontanus, claimed that these two authors had compiled a new table of planetary equations giving better results than the standard Alfonsine Tables, but this "new" table has not been found in any manuscript or printed edition.[51]

As already mentioned, the Alfonsine Tables were first printed in 1483 by Erhard Ratdolt in Venice. A few years later (1492) and in the same town, a second edition appeared, edited by Johannes Lucilius Santritter. The entries for the planetary equations are the same in both sets of tables but, in the second edition, the planets were inexplicably presented in the order Venus, Mercury, Mars, Jupiter, and Saturn (rather than in the usual order where Mercury precedes Venus). Also in the 1492 edition, the second column for the argument, displaying the complement in 360°, was eliminated; this left enough space on the page to include five extra columns for the differences between successive entries in the remaining columns.

In 1503 Petrus Liechtenstein printed another set of tables in Venice, the *Tabule astronomice Elisabeth Regine*. It was much less popular than the standard version of the Parisian Alfonsine Tables, but it is historically significant because in his *Commentariolus* Copernicus cited its author, Alfonso de Córdoba, who was in the service of Pope Alexander VI in Rome.[52] The tables for the planetary equations, as well as all the others in this set, depend on the Parisian Alfonsine Tables, both for models and parameters. However, this is not true for the presentation. First, for each planet the equation of centre is given in a different table from that for the equation of anomaly.

TABLE I. Alfonso de Córdoba's equation of centre of Mars (excerpt).

Longitude		Mars (°)	min
Leo 15	Leo 15	0; 0	60
Leo 20	Leo 10	0;55	60
Leo 25	Leo 5	1;49	59
…	…		
Sco 15	Tau 15	11;23	3
Sco 20	Tau 10	11;24	3
Sco 25	Tau 5	11;21	8
…	…		
Aqr 5	Aqr 25	2;13	58
Aqr 10	Aqr 20	1; 7	59
Aqr 15	Aqr 15	0; 0	60

As we have seen, John Vimond had also used this two-fold presentation, which was most uncommon in Latin astronomy,[53] but the two sets differ in several important aspects (see Table I). Second, the argument in the tables for the equation of centre is given at 5º-intervals (as is the case in the tables for the equation of anomaly), in contrast to the tables in the standard tradition (1º-intervals). But most important of all is the fact that the argument in the table for the equation of centre represents the mean longitude of the planet, $\bar{\lambda}$, that is, the mean argument of centre plus the longitude of the planet's apogee. Thus, the argument is shifted by a quantity that, in each case, corresponds to the longitude of the apogee (Leo 15º in the case of Mars). Again, the purpose is to facilitate calculation. In turn, the tables for the equation of anomaly display the usual columns of the tables in the standard tradition (cols. 1, 2, 5, 6, and 7).[54]

The tabular innovations developed to facilitate computation of the true longitude of the planets paved the way to a substantial increase in the number of almanacs in the fourteenth and fifteenth centuries. Although there are some earlier examples of this genre, the various new presentations of the tables for planetary equations (such as double argument tables) made almanacs much easier to compile. In turn, since almanacs and ephemerides display directly the true positions of the planets at successive times, the user did not have the difficult task of computing planetary equations; hence, they were very popular, for they could be used even by those who had not mastered all the subtleties of astronomy.[55]

Perhaps the most elaborate and influential almanac in the late Middle Ages was the *Almanach perpetuum*.[56] Its tables, together with a short explanatory text, were first printed in two editions (one in Latin and the other in Castilian) in Leiria, Portugal, in 1496. The tables were derived from a set of astronomical tables in Hebrew called *ha–Ḥibbur ha-gadol* (*The great composition*) compiled by Abraham Zacut of Salamanca (1452–1515).[57] As regards the positions of the planets, Zacut's work was compiled in the framework of the Parisian Alfonsine Tables with 1473 as epoch. For each planet it gives the true longitude, the true argument of centre, and the true argument of anomaly for several days in each month (sometimes daily) for periods as long as 125 years in the case of the longitude of Mercury. For these three quantities there is a total of more than 42,100 entries and, in each case, the sign, the degrees, and the minutes are specified. We have certainly come a long way from the tables for the planetary equations in the *Handy tables*!

Conclusion

As regards planetary equations, the standard tradition, going back to Ptolemy's *Handy tables*, survived at least until the first printed editions of the Parisian Alfonsine Tables. Ptolemy's underlying models and most of the parameters involved were rarely challenged from about the middle of the second century to the end of the fifteenth century. Only two parameters appearing in the tables were changed in that period, and John Vimond seems to have been the first astronomer to have used them in Latin Europe.[58] Vimond depended on material from the Iberian Peninsula, most likely of Arabic origin.

Astronomers in Latin Europe in the fourteenth and fifteenth centuries were actively engaged with this well-defined tradition, but they did not simply reproduce the tables and texts of their predecessors, and many of them developed innovative approaches to facilitate computational tasks, such as double argument tables, displaced tables, separated tables, or shifted variables. User-friendliness, rather than improvement of the models or enhancement of precision, was the driving force for most of the efforts developed by table-makers in the computation of planetary positions. Nevertheless, the *editio princeps* of the Parisian Alfonsine Tables did not incorporate any of the various new presentations. These innovations in presentation have only been recognized in recent years and, taken together, they indicate that astronomers in Latin Europe reached a high level of mathematical competence in the late Middle Ages.

REFERENCES

1. Unless otherwise specified, by planets we mean the five visible planets of Antiquity, although we are well aware that at the time the Sun and the Moon were also considered planets.
2. We do not treat tables in Islamic zijes systematically but, occasionally, we refer to some of them. For a survey of these zijes, see D. A. King and J. Samsó, with a contribution by B. R. Goldstein, "Astronomical handbooks and tables from the Islamic world (750–1900): An interim report", *Suhayl*, ii (2001), 9–105.
3. G. J. Toomer, *Ptolemy's Almagest* (New York and Berlin, 1984), 549–53.
4. See Toomer, *Almagest* (ref. 3), 548, n. 54.
5. O. Neugebauer, *A history of ancient mathematical astronomy* (Berlin, 1975), 183.
6. Although the equation of centre for the Sun and the equation in longitude for Venus are the same in the *Almagest* (Toomer, *Almagest* (ref. 3), 167, 552), their apogees differ: the solar apogee is 65;30° and tropically fixed (Toomer, *Almagest* (ref. 3), 155), whereas the apogee of Venus is 55° in Ptolemy's time and sidereally fixed, and thus subject to precession (Toomer, *Almagest* (ref. 3), 470; *cf*. Neugebauer, *Ancient astronomy* (ref. 5), 58, 154, 182).
7. W. D. Stahlman, "The astronomical tables of Codex Vaticanus Graecus 1291", unpublished Ph. D. thesis, Brown University, 1959 (University Microfilms no. 62-5761), 295–324. This dissertation includes an edition of Ptolemy's *Handy tables*. A. Tihon and R. Mercier are currently editing

the *Handy tables*; only two volumes have been published so far, and they do not deal with planetary equations.

8. See Neugebauer, *Ancient astronomy* (ref. 5), 183–6 (*Almagest*) and 1002–3 (*Handy tables*).
9. Stahlman, "Tables" (ref. 7), 307–12.
10. O. Neugebauer, *The astronomical tables of al-Khwārizmī* (Copenhagen, 1962).
11. In this tradition the apogees of Venus and the Sun are the same and both are sidereally fixed: see B. R. Goldstein and F. W. Sawyer, "Remarks on Ptolemy's equant model in Islamic astronomy", in *Prismata: Festschrift für Willy Hartner*, ed. by Y. Maeyama and W. G. Salzer (Wiesbaden, 1977), 165–81, p. 168.
12. C. A. Nallino, *Al-Battānī sive Albatenii Opus astronomicum* (2 vols, Milan, 1903–7), ii, 108–37.
13. See Goldstein and Sawyer, "Equant" (ref. 11), 168. Ḥabash identified both the eccentricities and the apogees of Venus and the Sun, despite the lack of justification based on observations or based on anything said by Ptolemy in the *Almagest* (or elsewhere). In modern terms, this would mean that the solar orb serves as the deferent for Venus; but this claim was not made by any medieval scholar. Nevertheless, the medieval tradition was to keep the apogee and eccentricity of Venus equal to those of the Sun, such that whenever the parameters for the Sun were changed, the same changes were applied to Venus.
14. See G. J. Toomer, "A survey of the Toledan Tables", *Osiris*, xv (1968), 5–174, p. 67.
15. Toomer, "Toledan Tables" (ref. 14), 60–8; and F. S. Pedersen, *The Toledan Tables: A review of the manuscripts and the textual versions with an edition* (Copenhagen, 2002), 1265–1306.
16. Toomer, *Almagest* (ref. 3), 588.
17. See Neugebauer, *Ancient astronomy* (ref. 5), 1005–6; and J. Chabás and B. R. Goldstein, *A survey of European astronomical tables in the late Middle Ages* (Leiden, 2012), 118.
18. The only known example of this kind of table where the entries are given at intervals of half a degree is preserved in a double folio now in the General Archive of Navarre: see J. Chabás, "The Toledan Tables in Castilian: Excerpts of the planetary equations", *Suhayl*, xi (2012), 179–88.
19. See J. Samsó and E. Millás, "The computation of planetary longtiudes in the *zīj* of Ibn al–Bannā'", *Arabic sciences and philosophy*, viii (1998), 259–86; reprinted in J. Samsó, *Astronomy and astrology in al-Andalus and the Maghrib* (Aldershot, 2007), Essay VIII.
20. A. Mestres, "Materials Andalusins en el Zīj d'Ibn Isḥāq al-Tūnisī", unpublished Ph.D. thesis, Universitat de Barcelona, 1999, 50–1 and 234–5.
21. J. Chabás and B. R. Goldstein, *The Alfonsine Tables of Toledo* (Dordrecht, 2003), 38–9 and 157–60.
22. J. Chabás and B. R. Goldstein, "Early Alfonsine astronomy in Paris: The tables of John Vimond (1320)", *Suhayl*, iv (2004), 207–94, pp. 236–56.
23. J. Chabás, "Were the Alfonsine Tables of Toledo first used by their authors?", *Centaurus*, xlv (2003), 142–50.
24. E. Poulle, "Jean de Murs et les tables alphonsines", *Archives d'histoire doctrinale et littéraire du moyen âge*, xlvii (1980), 241–71, p. 253.
25. Chabás and Goldstein, *Toledo* (ref. 21), 251–4.
26. Characteristic of the Parisian Alfonsine Tables is the consistent use of sexagesimal days and angles. Angles are given in physical signs of 60º (in contrast to zodiacal signs of 30°): an angle a,b means $a \cdot 60 + b$ degrees (where a and b are integers such that $0 \leq a \leq 5$ and $0 \leq b \leq 59$; and 6,0° = 360°). In our notation 10s 25º means $10 \cdot 30º + 25º = 325º$, that is, "s" signifies a zodiacal sign of 30º. Sexagesimal fractions of a degree are used in the same way with both physical signs and zodiacal signs.
27. For example, for a list of manuscripts extant in Spain or of Spanish origin, see Chabás and Goldstein, *Toledo* (ref. 21), 292–303.
28. Samsó and Millás, "Ibn al-Bannā'" (ref. 19), 268–70.
29. J. D. North, *Richard of Wallingford* (3 vols, Oxford, 1976), iii, 196.
30. Samsó and Millás, "Ibn al-Bannā'" (ref. 19), 278–85.

31. J. Chabás and B. R. Goldstein, "John of Murs's Tables of 1321", *Journal for the history of astronomy*, xl (2009), 297–320.
32. There were a few double argument tables in Islamic zijes prior to 1320, e.g., Ibn Yūnus (*c*. 990) had such a table for the lunar equations as did al-Baghdādī (*c*. 1285): see D. A. King, "A double-argument table for the lunar equation attributed to Ibn Yūnus", *Centaurus*, xviii (1974), 129–46, and C. Jensen, "The lunar theories of Al-Baghdādī", *Archive for history of exact sciences*, viii (1972), 321–8. Double argument tables for planetary latitudes attributed to Ibn al-Bayṭār, who is otherwise unknown, are preserved in Hyderabad, Andra Pradesh State Library, MS 298, Tables 66–77, where the horizontal headings are the true arguments of centre and the vertical headings are the true arguments of anomaly (both at intervals of 6°): see A. Mestres, "Maghribī astronomy in the 13th century: A description of manuscript Hyderabad Andra Pradesh State Library 298", in *From Baghdad to Barcelona: Studies in the Islamic exact sciences in honour of Prof. Juan Vernet*, ed. by J. Casulleras and J. Samsó (Barcelona, 1996), 383–443, p. 428. These tables are also cited in B. van Dalen, "Tables of planetary latitude in the *Huihui li* (II)", in *Current perspectives in the history of science in East Asia*, ed. by Y. S. Kim and F. Bray (Seoul, 1999), 316–29, p. 327. This manuscript contains the zij of Ibn Isḥāq (early 13th century), and Ibn al-Bayṭār is mentioned in chap. 18 of the canons to this zij. This implies that Ibn al-Bayṭār was active no later than the time of Ibn Isḥāq. A summary of chap. 18 appears in Mestres, *op. cit.*, 396–7. We are most grateful to van Dalen for sharing with us his translation of the Arabic text of chap. 18, the Arabic text of chap. 18 and of Tables 66–77, as well as his notes on this material.
33. E. Poulle, "John of Lignères", *Dictionary of scientific biography*, vii (1973), 122–8, pp. 123–4; and J. D. North "The Alfonsine Tables in England", in *Prismata: Festschrift für Willy Hartner*, ed. by Y. Maeyama and W. G. Salzer (Wiesbaden, 1977), 269–301, pp. 273–4, 278; reprinted in J. D. North, *The universal frame: Historical essays in astronomy, natural philosophy, and scientific method* (London, 1989), 327–59.
34. The reason is that in the range 150º–210º the entries for these two planets vary quite rapidly, and thus the accuracy of interpolation is increased by doubling the number of entries. Vimond was already aware of this rapid variation, and in his tables for the equation of anomaly for Mars and Venus, given at 3º-intervals, he displayed entries in the range 168º–192º at 2º-intervals.
35. Only three manuscripts containing theses tables are known. The other two use zodiacal signs of 30º (Erfurt, Biblioteca Amploniana, MS CA 2º 388, and London, British Library, MS add. 24070). All three have a column at the far right for the mean argument of anomaly from 180º to 360º, but the numbers in this column, and only in this one, are inverted in different ways in two manuscripts; for example, 234º is written as 54 3 (meaning 3,54°) in MS Lisbon; 24º 7s in MS Erfurt; and 7s 24º in MS London. It is possible that this reflects an archetype in Arabic or Hebrew, but see C. Burnett, "Why we read Arabic numerals backwards", in *Ancient and medieval traditions in the exact sciences*, ed. by P Suppes *et al*. (Stanford, 2000), reprinted in C. Burnett, *Numerals and arithmetic in the Middle Ages* (Aldershot, 2010), Essay VII, for some examples of this inversion in medieval Latin texts. Moreover, the three manuscripts differ in another aspect: MS Lisbon has a column for successive differences, MS Erfurt has no such column, and MS London has columns and rows for successive differences. This is definitely a good example of the intervention of copyists when transmitting the very same table, without altering its presentation or any of its essential features.
36. In the case of Mars, for instance, $c_5(90) = 2;28^\circ$, $c_6(90) = 33;22^\circ$, and $c_7(90) = 2;49^\circ$ in the standard tradition beginning in the *Handy tables* (see Table A). Thus, $c_6(90) - c_5(90) = 30;54^\circ$ and $c_6(90) + c_7(90) = 36;11^\circ$. The entry in John of Lignères's table of Mars for $\bar{\kappa} = 0^\circ$ and $\bar{\alpha} = 90^\circ$ is 30;54º and that for $\bar{\kappa} = 180^\circ$ and $\bar{\alpha} = 90^\circ$ is 36;11º.
37. For Venus, the maximum entry in John of Lignères's table of 1325 is 48;33º (at $\bar{\kappa}$ = 3,54º and 4,0º; and $\bar{\alpha}$ = 2,18º) as displayed in Table E. If $\bar{\kappa}$ = 4,0º = 240º, then $c_3(240) = 1;55^\circ$, using an equation of centre with a maximum of 2;10º (Vimond's value), and $c_4(240) = 31$. Thus, $\alpha = 138 - 1;55 = 136;5^\circ$. Therefore, $c_5(136;5) = 1;11^\circ$, $c_6(136;5) = 45;59^\circ$, and $c_7(136;5) = 1;16^\circ$. Finally, the combined equation is $1;55^\circ + 45;59^\circ + (1;16 \cdot 31/60) = 48;32^\circ$, in agreement with the entry.

However, when performing the same calculation using an equation of centre with a maximum of 1;59° (as in the Toledan Tables), one finds a combined equation of 48;19°. As we shall see in the computation that follows, the results are also unambiguous in the case of Jupiter. The maximum entry in John of Lignères's table is 17;1° (at $\bar{\kappa}$ = 4,24° and $\bar{\alpha}$ = 1,48°). If $\bar{\kappa}$ = 4,24° = 264°, then c_3(264) = 5;57°, which is the maximum equation of centre in Vimond's tables, and c_4(264) = 7. Thus, α = 108 – 5;57 = 102;3°. Therefore, c_5(102;3) = 0;29°, c_6(102;3) = 11;3°, and c_7(102;3) = 0;32°. Finally, the combined equation is 5;57° + 11;3° + (0;32 · 7/60) = 17;4°, very close to the entry in John of Lignères's *Tabule magne*. However, if we use a table with a maximum equation of centre of 5;15° (as in the Toledan Tables), one finds a combined equation of 16;21°.

38. For a description of this manuscript, see M.-M. Saby, "Les canons de Jean de Lignères sur les tables astronomiques de 1321", unpublished thesis, École Nationale des Chartes, Paris, 1987, 516–20. A summary appeared as "Les canons de Jean de Lignères sur les table astronomiques de 1321", *École Nationale des Chartes: Positions des thèses*, 1987, 183–90.
39. Chabás and Goldstein, *Toledo* (ref. 21), 278. The Tables of Toulouse are an adaptation of the Toledan.
40. For interpolation in double argument tables, see M. Husson, "Ways to read a table: Reading and interpolation techniques of early fourteenth-century double-argument tables", *Journal for the history of astronomy*, xliii (2012), 299–319.
41. These tables are only extant in Latin and Hebrew versions; the Latin version was composed by John of Dumpno in 1260 in Palermo and survives uniquely in Madrid, Biblioteca Nacional de España, MS 10023. See J. Chabás and B. R. Goldstein, "Andalusian astronomy: *al-Zīj al-Muqtabis* of Ibn al-Kammād", *Archive for history of exact sciences*, xlviii (1994), 1–41; and B. R. Goldstein, "Solomon Franco on the zero point for trepidation", *Suhayl*, x (2011), 77–83.
42. For the *Tabule anglicane* see J. D. North, "England" (ref. 33); for the Tables for 1321, see J. Chabás and B. R. Goldstein, "John of Murs" (ref. 31); for the *Tabulae permanentes*, see B. Porres and J. Chabás, "John of Murs's *Tabulae permanentes* for finding true syzygies", *Journal for the history of astronomy*, xxxii (2001), 63–72; for Immanuel ben Jacob Bonfils of Tarascon, see P. Solon, "The 'Hexapterygon' of Michael Chrysokokkes", unpublished Ph.D. thesis, Brown University, 1968 (Proquest, UMI, AAT 6910019), and P. Solon, "The Six Wings of Immanuel Bonfils and Michael Chrysokokkes", *Centaurus*, xv (1970), 1–20; for Levi ben Gerson, see B. R. Goldstein, *The astronomical tables of Levi ben Gerson* (Hamden, CT, 1974); for Juan Gil of Burgos and Joseph Ibn Waqār of Seville, see J. Chabás and B. R. Goldstein, "Computational astronomy: Five centuries of finding true syzygy", *Journal for the history of astronomy,* xxviii (1997), 93–105, pp. 94–6; for the Tables of Barcelona, see J. M. Millás, *Las Tablas Astronómicos del Rey Don Pedro el Ceremonioso* (Madrid and Barcelona, 1962), and J. Chabás, "Astronomia andalusí en Cataluña: Las Tablas de Barcelona", in *From Baghdad to Barcelona*, ed. by Casulleras and Samsó (ref. 32), 477–525; for Judah ben Asher II of Burgos, see B. R. Goldstein, "Abraham Zacut and the medieval Hebrew astronomical tradition", *Journal for the history of astronomy*, xxix (1998), 177–86, pp. 179–81.
43. For details, see J. Chabás and B. R. Goldstein, "Displaced tables in Latin: The Tables for the Seven Planets for 1340", *Archive for history of exact sciences*, lxvii (2013), 1–42. Note that the term 'displaced' applied to tables was coined by E. S. Kennedy in 1977 as a translation of the Arabic *waḍʿī* (see his "The Astronomical Tables of Ibn al-Aʿlam", *Journal for the history of Arabic science*, i (1977), 13–23, espec. p. 16).
44. Note that the *equatio centri* is always positive; it reaches a minimum of 0;3° at 72°–78°, and a maximum of 11;57° at 246°–252°. For the minutes of proportion there are two discontinuities (from 60′ to 0′ between 77° and 78°, and from 0′ to 60′ between 259° and 260°), to keep them positive in all cases.
45. Note that the equation of centre is negative between 0° and 180°, and positive between 180° and 360°; it reaches a minimum of –5;57° at 90°–96°, and a maximum of 5;57° at 264°–270°. The minutes of proportion are positive between 89° and 271°, and negative for the rest of the arguments.
46. J. Chabás, "From Toledo to Venice: The Alfonsine Tables of Prosdocimo de' Beldomandi", *Journal for the history of astronomy*, xxxviii (2007), 269–81.

47. J. Chabás and B. R. Goldstein, *The Astronomical Tables of Giovanni Bianchini* (Leiden, 2009).
48. For a list of manuscripts that contain Bianchini's tables, see Chabás and Goldstein, *Bianchini* (ref. 47), 14. The owners of manuscript copies of these tables include Johannes Regiomontanus (Nuremberg, Stadtbibliothek, Cent V 57) and Johannes Virdung (Vatican, Biblioteca Apostolica, MS Pal. lat. 1375).
49. J. Dobrzycki, "The Tabulae Resolutae", in *De astronomia Alphonsi Regis*, ed. by M. Comes, R. Puig, and J. Samsó (Barcelona, 1987), 71–7; J. Chabás, "Astronomy in Salamanca in the mid-fifteenth century: The *Tabulae resolutae*", *Journal for the history of astronomy*, xxix (1998),167–75; and J. Chabás, "The diffusion of the Alfonsine Tables: The case of the *Tabulae resolutae*", *Perspectives on science*, x (2002), 168–78.
50. B. Porres, "Les tables astronomiques de Jean de Gmunden: Édition et étude comparative", unpublished Ph.D. thesis, École pratique des hautes études, Section IV, Paris, 2003.
51. J. Dobrzycki and R. L. Kremer, "Peurbach and Marāgha astronomy? The ephemerides of Johannes Angelus and their implications", *Journal for the history of astronomy,* xxvii (1996), 187–237, pp. 187–8.
52. Alfonso de Córdoba, known as "Hispalensis", came from Seville (Latin: *Hispalis*) and his origin is well attested in various printed texts of the early sixteenth century. His place of origin is explicitly given as "patria hispalensis" and he is cited as "Alfonso hispalensi de Corduba". Hence there is no reason to emend the text to "hispaniensis", that is, from Spain, as has been suggested: see N. M. Swerdlow, "The derivation and first draft of the Copernicus's planetary theory: A translation of the Commentariolus with commentary", *Proceedings of the American Philosophical Society*, cxvii (1973), 423–512, pp. 451–2. Copernicus's *Commentariolus* is undated, but it is usually taken to be from about 1514. Alfonso dedicated his work to Queen Isabella of Castile and Aragon (1451–1504), whose name in Latin was Elisabeth. On this set of tables, see J. Chabás, "Astronomy for the court in the early sixteenth century: Alfonso de Córdoba and his Tabule astronomice Elisabeth Regine", *Archive for history of exact sciences*, lviii (2004), 183–217.
53. This separation was intended to distinguish clearly between the columns that depend on one variable from those that depend on the other. We know of another example of this two-fold presentation in Erfurt, Biblioteca Amploniana, MS Q 362 (ff. 28r–36r), also in the Alfonsine corpus, whose layout seems unrelated to those by either John Vimond or Alfonso de Córdoba.
54. The fact that the argument in the table for the equation of centre was chosen to be the mean longitude of the planet makes the table less useful in the long term, for it does not take into account the slow motion of the apogee due to precession.
55. An almanac lists successive true positions of each planet at intervals of one day or a few days, that is, each planet is listed separately; hence, the information for a given day is scattered among various tables. An ephemeris displays the true positions of all planets in a single row at intervals of one day or a few days for a certain number of years. That is, the difference between an almanac and an ephemeris is only one of presentation.
56. J. Chabás and B. R. Goldstein, *Astronomy in the Iberian Peninsula: Abraham Zacut and the transition from manuscript to print* (Transactions of the American Philosophical Society, xc/2; Philadelphia, 2000).
57. On the various editions of the *Almanach perpetuum* and its impact on the Jewish community as well as on Christian and Muslim scholars, see Chabás and Goldstein, *Zacut* (ref. 56), 161–71.
58. To be sure, the apogee of Venus was also changed, but this does not modify the table for the equations for this planet.