Hist. Sci., xl (2002)

HAVING A KNACK FOR THE NON-INTUITIVE: ARISTARCHUS'S HELIOCENTRISM THROUGH ARCHIMEDES'S GEOCENTRISM

Jean Christianidis, Dimitris Dialetis and Kostas Gavroglu
University of Athens

> Certain people, [propounding] what they consider a more persuasive view ... supposed the heavens to remain motionless, and the Earth to revolve from west to east about the same axis [as the heavens].... However, they do not realize that, although there is perhaps nothing in the celestial phenomena that would count against that hypothesis, at least from simpler considerations, nevertheless from what would occur here on earth and in the air, one can see that such a notion is quite ridiculous.[1]
>
> Ptolemy

INTRODUCTION

It is always interesting to read the analysis of rival viewpoints in the same text. But it is particularly intriguing when the author of the text happens to be championing one of the two rival viewpoints. To understand the status of ideas that advocated a non-stationary earth through the bible of geocentrism, the *Almagest*, may, perhaps, be asking too much. Yet Ptolemy appears to be a gracious adversary. He explicitly states that though on observational grounds there can be no objection to such an idea (even, perhaps, implicitly including heliocentrism), they should be denied on physical criteria. Elaborating in Book 1 of the *Almagest* all the physical reasons as to why the geocentric viewpoint is to be preferred, Ptolemy never again becomes involved in such ontological considerations while he unfolds the almost mesmerizing interactions among the motions of the epicycles on the deferents, which may be moving in eccentric circles, while having regular motions around the equants. But reading this passage there is a sense of *déjà vu*: "from simpler considerations", that is, on empirical grounds a(ny) motion of the Earth, is, *of course*, unacceptable. And trivially so. Hence, if any ideas involving the motion of the earth had been entertained by some for sometime, it should have been part of an overall context where the empirical or the philosophical considerations would not have been as constraining a factor, as they are required to be in the *Almagest*. What, then, were the characteristics of this context? Within what discourse was such a ludicrous notion as the motion of the earth entertained? How did such a manifestly anti-empirical and obviously counterintuitive notion become a legitimate alternative to geocentrism?

"There is not the slightest doubt that Aristarchus was the first to put forward the heliocentric hypothesis. Ancient testimony is unanimous on this point." This is how Sir Thomas L. Heath starts his principal chapter on Aristarchus of Samos

0073-2753/02/4002-0147/$2.50

in his classic work of the same title.[2] Is this really the main conclusion that can be arrived at? Ancient testimony is indeed unanimous on this point. But does such unanimity lead *necessarily* to the fact that Aristarchus was the first to have claimed the heliocentric hypothesis? We think not. Aristarchus, it appears, was one of the prominent astronomers and mathematicians who had *further* elaborated the implications of a non-geocentric and non-geostatic view — itself a view with a long tradition — by explicitly considering a particular version of this view that was the heliocentric view. In this paper we argue that the unanimity of the sources is a concurrence reflecting the *legitimacy* of such a version and its *legitimation* through the establishment of a new mathematical practice which led to the coming-to-be of trigonometry and the emergence of the processes for 'arithmetizing' geometry.[3] We argue that during the Hellenistic period and for a relatively short period, the description of the cosmos appears to have been relatively free of philosophical constraints.

There has been an impressive corpus of scholarship concerning Archimedes's famous passage from the *Sand-reckoner* where we receive the main information about Aristarchus's heliocentrism.[4] As a result there has been a deep insight into what Aristarchus may have claimed. But no one seems to have examined Archimedes's views about Aristarchus's heliocentrism, which may be implied through Archimedes's description of Aristarchus's heliocentrism. For example, it is quite remarkable that Archimedes uses the geocentric viewpoint interchangeably with the heliocentric and that such a use has not been noted by those who discussed Aristarchus's heliocentrism. It can also be argued that both Aristarchus's heliocentrism and the lack of its criticism not only by Archimedes but also in the passages we have from much later,[5] is suggestive of the possibilities provided by a new mathematical culture in the making, during a period from the publication of Euclid's *Elements* (*c*. 300 B.C.) to the middle of the second century B.C. when Hipparchus was active. With Aristarchus and Archimedes we have the beginnings of a new mathematical practice differentiating itself from the Euclidean style. The quantitative dimension began to be essential in mathematical manipulations. It now became possible to pose geometrical problems in such a way that they were amenable to numerical solutions, and it also became possible in astronomy to estimate quantities by using geometrical methods in conjunction with data provided by direct observations — even though we do not know the exact details of how they made the observations. Such a new practice was accompanied by changes in the intellectual milieu whereby the ontological commitments related to these quantities were becoming progressively less important as a criterion for the soundness of the hypotheses used and the validity of the results derived by using these hypotheses. It is within such a framework of a new practice and a new intellectual milieu that, we believe, a space was created for discussing and elaborating a non-intuitive hypothesis such as heliocentrism. Aristarchus appears to have been among the most prestigious defenders of heliocentrism, and, perhaps, also among the last defenders of a long tradition in which the motion of the earth was advocated and different scenarios were advanced concerning the centre around which the earth moved. In other words, from all the available sources it seems that *on the whole* the alternatives to geocentrism, far from being considered as heretical and downright wrong, were entertained as viable alternative mathematical modes for understanding astronomical phenomena. It is only when physical and philosophical considerations became the almost exclusive criterion for the legitimacy of the mathematical description, that geocentrism was preferred to heliocentrism, despite the latter's advantages in astronomical terms as Ptolemy, the geocentrist *par excellence*, is ready to admit.

AN OLD IDEA: THE MOTION OF THE EARTH

Until the circulation of the *Almagest* in the second century A.D., the geocentric and geostatic view with the concentric spheres, though the dominant model for the universe, was not the only one. The very fact that there was more than one alternative to what appeared intuitively natural is, in itself, a thought-provoking issue. That these alternatives had to be taken seriously by their adversaries and were scrutinized not with respect to astronomical data but with respect to criteria based purely on intuitive considerations is an indication that they comprised a framework of ideas that for a certain audience were rather persuasive. Those who did not agree with them continued to devise counterarguments, refusing to accept that the case had been closed. It is difficult to suppose that philosophers and astronomers would continue to be preoccupied for such a long period with ideas that did not have a following, and corroborating astronomical evidence of some form.

Many of the historians who have discussed Aristarchus's heliocentrism have repeatedly expressed the view that any idea other than the geocentric and geostatic views of the universe must somehow have been a marginal consideration that was not taken seriously.[6] There is no supporting evidence from the extant sources for such an assessment. If anything, it appears that for two or three centuries the idea of a moving earth was an alternative that could not easily be dismissed. Belief in a moving earth was usual among the Pythagoreans. Philolaus (last decades of the fifth century), one of their better known figures, had asserted that *both* the sun *and* the earth moved around a central fire, though we are not sure of the exact details of these motions. This system could account for the apparent rotation of the heavens, but it could not account for the apparent irregularities among the planetary motions, though this was not regarded as a defect as these irregularities had not yet been systematically observed. Interestingly, it was the inability to observe the central fire that was deemed to be its weak point, rather than the movement of the earth. Extant sources refer to Hicetas, another Pythagorean from Syracuse, who "holds that the heaven, the sun, the moon, the stars, and in fact things in the sky remain still, and nothing else in the universe moves except the earth; but, as the earth turns and twists about its axis with extreme swiftness, all the same results follow as if the earth were still and the heaven moved".[7] Hicetas did not include the central fire in his system and his conception was based on there being one moving body, the earth, while all the other bodies, including the sun, stood still. Another Pythagorean, Ecphantus of Syracuse (fifth century B.C.), is said to have promoted the same idea of a moving-rotating earth. Hence, by the first half of the fourth century, some members of the Pythagorean School had already abandoned the doctrine of a central fire and had proposed cosmic systems based on the idea of a moving and/or rotating earth. It appears that a certain type of motion of the earth was sufficient for them to provide a qualitative description of the cosmos. Nevertheless one should also bear in mind that the Pythagorean views on the possible motion of the earth were formed within a context in which, as Goldstein and Bowen note, "the explanandum in their theories is not so much a physical phenomenon as the ethical and aesthetic order it supposedly exhibits".[8]

Historians, on the whole, agree that Heraclides of Pontus (*c.* 388 – *c.* 310 B.C.) was the first to have formulated the doctrine that the earth rotates on its axis from west to east. He was born at Heraclea in Pontus, and emigrated to Athens where he became a pupil of Speusippus, and afterwards possibly of Plato himself. He is said to have attended the Pythagorean School and he also seems to have received instruction from Aristotle. Furthermore, we read in Simplicius:

> Hence we actually find a certain person, Heraclides of Pontus, coming forward (παρελθών) and saying that, even on the assumption that the earth moves in a certain way, while the sun is in a certain way at rest, the apparent irregularity with reference to the sun can be saved.[9]

This passage has presented a number of philological problems that do not concern us here.[10] What is important for us is that Simplicius makes it clear that Heraclides (or whoever had defended this view) had elaborated a radically different way of saving the phenomena by abandoning the usual idea of the earth being at rest. But we note another aspect of this passage. The word παρελθών implies that Heraclides (or whoever had defended this view) came forward and spoke to some kind of an assembly. It indicates that there was an audience, and we know that in Athens at the time of Heraclides the audiences were neither passive nor did they gather for formalistic reasons. The role of the audiences in legitimizing new ideas does not appear to be a characteristic confined to modern times.

The issue of the earth's movement has been the subject of a number of studies on Plato and Aristotle. Although it is definitely the case that both argued for a geocentric and geostatic cosmos, it is equally true that in part their arguments were also aimed at counteracting contemporary views that were sympathetic towards some kind of a motion of the earth. Plato himself does not appear to have been unambiguous on the question of the motion of the earth, and we have mentioned that it is said that in his old age he had become sympathetic towards the view that assumed a moving earth. It is not important for our argument to see whether this was actually the case. It is, however, important to note that the testimony comes from Plutarch, the biographer and philosopher of the first century A.D. who was particularly sympathetic towards Platonic views. Plutarch writes: "Theophrastus also adds that Plato in his old age regretted that he had given the earth the middle place in the universe, which was not appropriate to it."[11] This is a sentence coming immediately after a detailed reference to Aristarchus's heliocentrism, starting with the question of whether "Plato put the earth in motion".[12] And the testimony is expressed in a straightforward way without any hint of embarrassment that Plato may have entertained such views. Furthermore, Aristotle notes that "many others too agree that the place in the centre should not be assigned to the earth".[13] There are convincing arguments that the persons to whom Aristotle refers were not Pythagoreans but some of his own contemporaries.[14] It should be emphasized that Aristotle's criticism of the ideas referring to a motion of the earth are based on the intuitive considerations of an observer whose criterion is the "physics" of earthly phenomena. Certainly, he does not choose to measure these ideas against heavenly phenomena.

Though Aristotle's considerations were philosophically consistent, the alternatives could not be dismissed on astronomical grounds. Scholars who have studied Plato systematically have not reached a consensus over whether or not Plato may have entertained views sympathetic to some motion of the earth. Furthermore, it is agreed that Heraclides formulated his viewpoint *after* Aristotle wrote *On the heavens*. To have such a viewpoint propounded immediately after *On the heavens* is not necessarily a tribute to the ingenuity of an isolated scholar, but rather an indication of the existence of a receptive audience for such ideas. The possibility of the motion of the earth is an idea that simply would not go away, quite apart from philosophical considerations. Whether it was because astronomical observations were more readily accommodated within such an idea, or whether, later, it was because there was an audience with a critical attitude towards Aristotelian doctrines, is a secondary issue. What is important is to realize that the persistence of this particular viewpoint for so long a period could not have been the result of zealous behaviour on the part of some, but rather it was the result of the persuasiveness of the arguments, coupled with the existence of some kind of corroborating evidence and of a receptive audience. A number of not insignificant writers continued to be intrigued with the issues surrounding the motion of the earth. Until the time of Ptolemy, it appears that geocentrism was far from commanding a general consensus. Seneca's view on the matter, when he said that they should clarify whether God moves the heavens or the earth, is particularly significant since he lived in a period so close to that of Ptolemy.[15]

It should therefore be noted that by the time of Aristarchus and Archimedes, there seems to have been a decline in belief in, and an intense questioning of, the *only* mathematical model for the cosmos. Nevertheless, and this is most important, what became well established was "the turning of astronomy into a mathematical science, that is, a deductive mathematical explanation of physical phenomena".[16] Any new ideas would continue to be formulated within this mathematical framework, but also within a conceptual framework with many more degrees of freedom compared to the constraints of Aristotelian physics, since in this transitory period there was no convincing model of the cosmos and no compelling reasons to adhere exclusively to Eudoxus's concentric sphere model.

ARCHIMEDES THROUGH ARISTARCHUS

We would like to read the extant source material on the assumption that what is extant is also a reflection of what is not extant. This is not to say that extant works tell us about the specific content of lost works — though this may, indeed, happen in a number of instances. The point we are making is that the extant sources express to a certain degree the ambience of the corpus that existed and has been lost and whose specific content we do not know; and, yet, we know that this corpus reflected the way people chose to state their opinions and construct their arguments. Existing works can be read in order to discern the context formed by *both* what has survived *and* what has been lost. To conjecture the content of what has been lost through an analysis of what is extant has been part of the standard analysis. But such a search for absences should also be complemented by an attempt to assess the status of what exists within the context determined by what has been lost inasfar as this can be surmized and inferred through what exists. This is how we intend to read our sources.

By far the most important information about Aristarchus's heliocentrism comes from the passage in the *Sand-reckoner* where Archimedes gives his own account of Aristarchus's ideas. This is a short tract where Archimedes, addressing King Gelon,[17] proceeds to calculate the number of specks of sand that could fill the universe in his attempt to calculate and express a very large number (10^{63}, much larger than 10^4 (myriad), the largest number for which the Greeks had a distinct symbol). Traditionally, the *Sand-reckoner* was considered to be among the very late works of Archimedes, a composition of his mature age. But in his meticulous study on the chronological ordering of the entire Archimedean corpus, W. R. Knorr has argued persuasively for an early dating of the *Sand-reckoner* on the ground that it seems better associated with the science of the earlier part of the third century than with that of the later part.[18] It might therefore be that Archimedes wrote his account of Aristarchus's heliocentric views at a date much closer to when Aristarchus may have defended his heliocentrism.[19] Commonly, Aristarchus's lifetime is placed in the last decades of the fourth and the first decades of the third century B.C., given that he was a pupil of Strato of Lampsakos, presumably at Alexandria, before the latter's assumption of the headship of the Peripatetic School in Athens, in 287 B.C., and that he had, according to Ptolemy, observed the summer solstice in 280 B.C.[20] Let us quote the relevant passage.[21]

> You know [King Gelon] that most astronomers designate by the word 'cosmos' the sphere whose centre coincides with the centre of the earth and whose radius is equal to the straight line connecting the centre of the sun and the centre of the earth; for you have gathered such information from the representations in their

> writings. But Aristarchus of Samos published in writing certain hypotheses, in which it follows, from the assumptions made, that the cosmos must be many times greater than the one mentioned before. He assumes namely that the fixed stars and the sun remain unmoved, while the earth moves round the sun through the circumference of a circle, which lies in the midst of the course; and that the sphere of the fixed stars, situated about the same centre as the sun, is so great that the circle in which the earth is supposed to revolve, has the same ratio to the distance of the fixed stars as the centre of a sphere to its surface. Quite obviously, this is impossible; for since the centre of a sphere has no magnitude, it cannot be conceived to bear any ratio to the surface of the sphere. We must, however, take Aristarchus to mean this: since we conceive the earth to be, as it were, the centre of the world, the ratio that the earth bears to what we call the cosmos is the same as the ratio that the sphere containing the circle in which the earth is conceived to revolve bears to the sphere of the fixed stars. For he adapts the proofs of the phenomena to an hypothesis of this kind, and in particular he appears to suppose the magnitude of the sphere in which he makes the earth move to be equal to what we call the cosmos.... Aristarchus tried to prove that the diameter of the sun is greater than 18 times, but less than 20 times, the diameter of the moon. But I go even further than Aristarchus, in order that the truth of my proposition may be established beyond dispute, and I suppose that the diameter of the sun to be about 30 times that of the moon and not greater.[22]

What Archimedes does, in the more technical sense, is the following. The proportion suggested by Aristarchus is

radius of the earth's orbit : radius of the sphere of fixed stars = centre : surface,

and Archimedes substitutes for the problematic second ratio,

radius of the earth's body : radius of the sun's orbit.

All historians of science who have discussed this passage have concentrated on understanding through Archimedes's account, the details of Aristarchus's heliocentrism. But none has raised what amounts to a complementary question: What do we understand to be Archimedes's *assessment* of Aristarchus's views? What further insights can we derive about Aristarchus's heliocentrism, by trying to understand what Archimedes 'feels' about it, concentrating on the way he chose to narrate it? Attempting to answer these questions, we are led to some interesting answers.

Firstly, and in contrast to what is the received view, we argue that Archimedes is in no way critical of the heliocentric hypothesis itself.

Secondly, Archimedes appears to be using the geocentric view and the heliocentric view interchangeably.

Thirdly, what Archimedes does is properly to correct Aristarchus's proportions, without undermining the latter's heliocentrism and almost wishing to save it!

The first point we are making is almost self-evident. There is not a single word to imply criticism by Archimedes of Aristarchus's sun-centred hypothesis. In fact, Archimedes wants an arbitrarily large but finite universe. Aristarchus had almost provided him with one, but it was, alas, in strict mathematical terms, an indeterminate one. Why should Archimedes have become involved with the arbitrary task of *transforming* Aristarchus's indeterminate universe into one which could be measurable, when the easiest and mathematically most expedient step would have been the denial of the heliocentric hypothesis that *led* to the rather ludicrous expression for the size of the universe in the first place? Why did he proceed to the transformation, when by rejecting heliocentrism, he could have guaranteed that the universe was indeed finite and that the indeterminacy of the expression was but the outcome of the un-naturalness of heliocentrism? Why does he proceed, instead, to commit himself to an expression that is mathematically well-defined but whose *derivation* leaves a lot to be desired? Why does he, after *correcting* Aristarchus's estimate, not make use of the new result, but proceed to make it larger in an arbitrary way, when he could have proposed the arbitrary size in the first place? Archimedes's unwillingness to touch the heliocentric hypothesis itself is not to be interpreted as an indication of Archimedes's belief in heliocentrism. Such a strong assertion is not warranted by the text and it needs further corroborating evidence, which we do not think can be found in other parts of the *Sand-reckoner* or in Archimedes's other texts. In fact, Archimedes seems to be indifferent over the issue of geocentrism or heliocentrism. Our point, then, is that Archimedes's unwillingness to deal in any manner with the heliocentric hypothesis could be an indication of his acceptance of heliocentrism as a legitimate (mathematical) hypothesis whose use can facilitate the understanding of the celestial phenomena, almost on a par with geocentrism.

Our first point, then, is this. Reading the key passage in Archimedes (whose significance outweighs all the other references to Aristarchus's heliocentrism[23]), there is nothing even remotely to suggest that Aristarchus's heliocentrism as such was either objected to or considered as a particularly daring proposal. In fact, Archimedes does not criticize heliocentrism itself, but, instead, he attempts to correct a rather technical point which, one may think, Archimedes feels may be undermining the original heliocentric hypothesis! Let us not forget that Archimedes was not only the mathematical genius he was, but also an astronomically sophisticated mathematician. He was the son of the astronomer Pheidias, the *Sand-reckoner* includes a lot of arithmetic and a lot of astronomy, Cicero tells us that he had constructed a planetarium, Hipparchus (through Ptolemy) mentions his determination of the length of the year, Macrobius reports on his theory concerning the mutual distances of the sun, the moon, and the planets, and, finally, the *Sand-reckoner* bears witness to his astronomical observations. Has it not been the case throughout the ages that the best choose as their discussants the most prestigious? And may not the decision to correct Aristarchus be an indication of the latter's prestige?

Why should Archimedes have troubled to bring in the geocentric hypothesis in order to make the implications of the heliocentric hypothesis more accessible to King Gelon, if the two hypotheses were antagonistic and contradictory to each other? And this is our second point: the particular words Archimedes uses in order to communicate to King Gelon some of the intricacies of the heliocentric hypothesis, aim at explaining something that is less well known in terms of something more familiar: "We must, however, take Aristarchus to mean this: since we conceive the earth to be, as it were, the centre of the world...." The words and phrasing used in the original passage are rather non-committal as to what Archimedes believes. Archimedes uses a didactic trick and his reference to geocentrism within the overall context of the passage is by no means to be taken as expressing Archimedes's ontological commitment. It is merely a methodological preference to help Archimedes make mathematically respectable Aristarchus's expression for the size of the universe. Our assessment that Archimedes is at ease with such an equivalence between the two hypotheses is further strengthened by what he actually does a little further on in the passage. He proposes to substitute the problematic proportion of Aristarchus

radius of the earth's orbit : radius of the sphere of fixed stars = centre : surface

by

radius of the earth's orbit : radius of the sphere of fixed stars =
radius of the earth's body : radius of the sun's orbit.

Archimedes changes the second ratio *while the first ratio is left unaltered*. In other words, what Archimedes does is to take a proportion that had been derived by Aristarchus through his heliocentrism, and to change one side of it by substituting an expression *derived* from the geocentric hypothesis, leaving the other side intact. In fact, the new proportion is one that involves measurable quantities, one belonging to the heliocentric system and the other to the geocentric. But in addition the proportion itself is the product of the heliocentric hypothesis. This, to our mind, is neither sloppy mathematics nor handwaving, but rather the expression of a new mathematical culture in the making during this period, of which more later. Far from being aggressive, Archimedes resembles a referee of an article who is particularly sympathetic to the author's agenda and suggests how to correct a "blunder" in the text!

Hence — and we now come to our third point — Archimedes does not appear to be aiming at undermining Aristarchus's viewpoint, but rather at showing that Aristarchus's viewpoint could continue to be considered as legitimate, despite a technical problem, for which Archimedes provides a solution. It appears as if Archimedes is concerned with "saving Aristarchus". If we do not take into consideration this aspect of Archimedes, his whole approach appears to be rather peculiar. Archimedes's insistence on discussing in such detail the ensuing proportion is an indication of his unwillingness to dismiss heliocentrism and his willingness to devise convincing arguments in order to derive a finite expression for the size of the universe. The fact that Archimedes uses no derogatory expression against Aristarchus, and his mentioning of Aristarchus in other passages of the *Sand-reckoner*, show a respect to one who is both the writer of the *On the sizes* and the explorer of heliocentrism.

But, perhaps, such was Archimedes's style when he was critical of others. In other words, it could be the case that Archimedes does not explicitly criticize people, but he intervenes in their results in such ways as to annul what others have been arguing. To check whether he systematically used such a *modus operandi*, we tried to see what the style of Archimedes's criticism of others is. We could find no explicit criticism by him of other people's work. We then tried to see in what context he mentions other persons' names. And, there, we met with a pleasant surprise. Archimedes, it appears, mentions names whenever he wants to praise someone's work, and he avoids criticism altogether. And the name most frequently mentioned by him in all of his works is Aristarchus!

Archimedes mentions Aristarchus by name ten times. All references are in the *Sand-reckoner*. He mentions him more times than Conon for whom he expresses his admiration at every opportunity. The latter is mentioned eight times, three times in the introduction of *On spirals*, twice in the introductions of the two books *On the sphere and the cylinder*, and three times in the *Measurement of a circle*. Eudoxus is mentioned four times: once in *The method of mechanical theorems*, once in the *Sand-reckoner*, and twice in the introduction in *On the sphere and cylinder I*. Democritus is mentioned once in *The method of mechanical theorems*. Eratosthenes is mentioned once, in the introduction of *The method of mechanical theorems*. Euclid is mentioned once in *On the sphere and cylinder I*, but it may, indeed, be the case that it is a phrase inserted by a scholiast, since Archimedes uses often theorems from Euclid's *Elements* and other works without mentioning his name. And the astronomer Pheidias, who was Archimedes's father, is also mentioned once, in the *Sand-reckoner*. Let us note that everyone who is mentioned is praised for his achievements, or else through the context it becomes clear that Archimedes shares the mathematical concerns and solicitudes of these people; and this is particularly important if we have in mind what Reviel Netz notes about Archimedes's being particularly keen to follow the work of other mathematicians.[24] So his reference to Aristarchus has an added significance.

To recapitulate, let us start from what Archimedes had achieved in the *Sand-reckoner*. He showed that it is possible to calculate an extremely large number, implying that is possible to construct an *arbitrarily* large number — for he chose to base his calculation on a size of the universe that is arbitrary anyway. Now the text is addressed to King Gelon and its tone is that of a work of popularization. The method he uses is to find the specks of sand which could fill the universe. For that, of course, he needs a finite universe — whether "small" or "large" does not really matter, as long as it is finite. What would be more natural than to ignore Aristarchus altogether and postulate a particularly large universe? Or take any of the existing estimates for finite universes? Why decide to start the story by choosing a mathematically problematic expression and, then, propose a well-defined expression in a mathematically sloppy way? One may even wonder why Archimedes decided to choose among all the estimates for the size of the universe the only available "infinite estimate" and decide to go through this roundabout way which lacks mathematical rigour in order to make it finite?

One answer may be that the author of the *Sand-reckoner* is not Archimedes. This is the easy way out, and not at all convincing. It is one of the reasons that led Erchardt and Erchardt-Siebold to question the authorship of the *Sand-reckoner*.[25] But if the authors felt that Archimedes is above such lack of economy or that he could not have adopted such a roundabout way, why do they feel that the anonymous author had such an unnatural way of thinking and writing? You need not be a mathematical genius to avoid handwaving. A conscientious mathematician has all the prerequisites to behave 'properly'. It may need talent to be strictly rigorous, but it only needs common sense to avoid being sloppy. Alternatively, it can be argued that Archimedes is very consciously following this particular approach in order to underline the fact that he chooses to deal with quantities that have been the result of calculations through this new discourse that Aristarchus must have been the first to have elaborated. He is not interested in using the "estimates" that have been derived as a result of rather metaphysical or muddled thinking. He is not even interested in arbitrarily postulating himself a finite size for the universe. It is to him much more preferable to start from an expression that is awkward for his purposes and not easily manageable, but which is the *outcome of this new way of doing mathematics and astronomy*. How justified are we in claiming that this seemingly warped and twisted way is an indication of Archimedes's commitment to a particular mathematical practice? We think quite a lot.

PRACTITIONERS OF A NEW DISCOURSE

From the above we are led to pose another question: Could it be the case that Archimedes and Aristarchus were part of a common enterprise, wishing to establish a new practice and sharing the conviction as to its strategic significance? Might it have been the case that they both adhered to (creating) a particular tradition and thus the common aims (and language) they shared was of far greater consequence than the differences between geocentrism and heliocentrism as hypotheses to be used for constructing mathematical theories? And might it have been the case that the rapport between Archimedes and Aristarchus that we have noted above may have been the expression of respect between a self-consciously great mathematician towards another of the same kind, independent of the (minor technical) differences? We shall try to argue why an affirmative answer to these questions is quite plausible.

Before doing so, however, we would like to note that in the period of Aristarchus and Archimedes favourable conditions began to form for such an enterprise. We have it from Simplicius that Sosigenes (second century A.D.) had analytically discussed the reasons as to what caused the model of Eudoxus (and its modifications by Callippus) progressively to lose the prestige it enjoyed.[26] The observation of the apparent brightness and/or the apparent diameter of the planets, especially of Venus, Mars and the moon, in addition to the systematic observations of the solar eclipses which are sometimes total and sometimes annular, led to the tentative conclusion about the change in the distances of the planets with respect to the earth. To these difficulties were added the change of the apparent velocity of the planets since they traverse different distances on the celestial sphere in equal times. In fact, according to Sosigenes, Aristotle, during the last years of his life, had in his *Physical problems* (which has been lost) expressed reservations concerning the model of the concentric spheres, since, it seems, the problem of the variation of the distances of the planets with respect to the earth could not be treated within the framework of Eudoxus's mathematical model and Aristotle's cosmological views. Sosigenes further asserts that the first doubts about the correctness of the model of concentric spheres were due to a student of Eudoxus, Polemarchus of Cyzikus (fourth century B.C.), who nevertheless tried to defend the model. The first person who appears to have tried — unsuccessfully — to propose an alternative solution was Autolycus of Pitane (*c*. 320 B.C.). Goldstein and Bowen consider his criticisms as the most influential and, as a result, they note that "there is no evidence of any adherence to homocentric models by ancient astronomers after him".[27]

Let us be reminded of some of the dates: Aristarchus *c.* 280, Archimedes *c.* 250, Eratosthenes *c.* 230, Apollonius *c.* 200, Seleucus *c.* 180, Hipparchus *c.* 130 B.C. These men form a continuity without any large gaps among themselves and, hence, what each knows of what someone before him claimed or did is quite reliable. It is, furthermore, the period that starts right after the *Elements* of Euclid. There has already been a systemization of the skies. There appear mathematicians who, having already mastered Euclidean geometry, attempt to approach the various problems by trying to devise and incorporate quantitative-arithmetical treatments into the geometry. It appears to be a period when mathematicians venture to draw up the framework for the application of Euclidean geometry. One way was the use of observationally measured quantities in order to calculate other astronomical quantities and proceed to the first quantitative models of the cosmos. Eudoxus's and Callippus's spheres were ingenious constructions, yet unable to respond to the quantitative needs of the new period. The first such work we know that is a sample of a quantitative model is Aristarchus's *On the sizes and distances of the sun and moon.* It is a treatise where distances and sizes are shown to be derivable as a result of geometrical methods and astronomical observations.

During this early Hellenistic period many strategies are framed and suggested. From earlier times, as is amply attested by Heraclides's assertions, there is the gradual emergence of the autonomy of the proposed hypotheses with respect to their physical meaning, if their implications could provide an additional insight into the phenomena. This autonomy is further entrenched in the newly formed context by the attempts at quantification. And hypotheses are legitimized by their *effectiveness in the* *enterprise of quantification*, whereas their questionable ontological implications do not necessarily undermine them. Furthermore, all the men we have mentioned are in geographical locations far removed from Athens, and so Aristotle's philosophical dominance may not have been as intense. It is also the period when Aristotle's successors at the Lyceum are rather critical of many aspects of Aristotelianism, and Aristarchus was the student of Strato — perhaps the most scathing critic of Aristotle before the Stoics.

So we are speaking of a period where the task of further quantification and arithmetization of geometry is gradually becoming one of the salient characteristics of mathematics. Greek geometry up to Euclid's time was not arithmetized to any remarkable extent. The issues concerning the notion of arithmetization in ancient Greek mathematics are particularly intriguing and rather intricate. David Fowler has systematically studied these issues, especially for the mathematics of the pre-Hellenistic period, and his analysis has led him to the conclusion that arithmetized mathematics considered, in effect, as the manipulation of fractions, were not developed before the second century B.C. and, significantly, cannot be found in the works of Archimedes. Our emphasis on the beginnings of a style inaugurated by Aristarchus and Archimedes and based on quantification and arithmetization of geometry, is by no means an alternative to Fowler's conclusions, but some comments may be useful. Firstly, the transition from the non-arithmetized to arithmetized mathematics could have not been a well-defined and abrupt *transition* either in intellectual terms or in terms of the time period when this new mathematical culture appeared. It is certainly the case that the period we are examining displays a host of particularities characteristic of such transition periods. Secondly, although Fowler considers Aristarchus's *On the sizes and distances* to be an "excellent illustration of what I have called 'non-arithmetized' mathematics", he finds in the same treatise "two mild exceptions to the statement I made".[28] Another exception, according to Fowler, is Archimedes's *On the measurement of a circle*. It is such fluctuations in style or ambivalent attitudes concerning the commitment to a particular way of dealing with ratios that are quite indicative of the transition period. Thirdly, if in Aristarchus's treatise we find evidence of those "techniques" whose further elaboration will, eventually, lead to the emergence of trigonometry, then it may not be surprising to find statements and approximations expressing differentiations from the dominant *problématique* of the treatise concerning the possible manipulation of ratios. Thus, the study of Aristarchus's *On the sizes and distances* appears to be absolutely critical in the discussion concerning the process of arithmetization, since it may be the first text whose genre cannot be unambiguously defined. It is, certainly, a mathematical tract, it is definitely a book of applied geometry, but, equally significantly, it is also a text of astronomy. It is, furthermore, the case that the issues surrounding arithmetization can be further clarified by including the discussion of computational as well as theoretical astronomy, especially since astronomy provides a more flexible framework of constraints when compared to other fields of applied geometry, and, hence, permits bolder manipulations. But the study of astronomy in clarifying issues of arithmetization is further strengthened by the acknowledged importance of Babylonian astronomy for the understanding of the later mathematics, especially in the convincing arguments of Babylonian influence in Aristarchus,[29] and even of Meton of the fifth century.[30]

The strengthening of the quantifying mathematical techniques gives for the first time the opportunity for the construction of a new discourse: this discourse involves mathematical modelling, and the insertion of parameters that could be measured or calculated. The development of such an activity becomes necessarily interwoven with the plurality of physical or other hypotheses. As in all cases the issue of the legitimacy of such an enterprise needs time and the training of persons who would adopt the new practice. Above all, the establishment of such a new culture needs persuasiveness; it needs social consent and concurrence. Underlining one particular conclusion of Reviel Netz, its pioneers must have relied less on proofs than on rhetoric.[31] And, if we judge by the names of the people involved in this period of less than 150 years, the results are most impressive. But there is an additional aspect worth noting. It is the question of the varying character of the specific demands we have from theories and models. The metamorphoses of our demands are in effect the changing relationship among the hypotheses, the mathematical constructs and the observed quantities. This changing relationship is reflected in the answers to questions like: How real are the mathematics being used? How close to reality are the mechanisms suggested by the proposed models? Which are the observables with which the calculations will have to agree? What, in fact, is meant by agreement with observations? How close should the agreement be in order to be considered as corroborating evidence? Though these are questions that have been raised throughout the ages, neither their relative significance for theory building nor the answers being formulated in one period could be considered as valid in another. This means that modes for the explanation of the phenomena have gone through deep changes themselves.

Surely what is extant from Aristarchus and what we learn about his heliocentrism from Archimedes, in addition to how Archimedes uses this hypothesis, *was not the development of methods akin to quantitative predictions concerning the planetary motions.* Aristarchus and Archimedes tried to quantify parameters of the cosmos: the distance of the moon, the distance of the sun, the sizes of the earth, the sun and the moon, and so on. The new program they were inaugurating appears to be a metric one for the cosmos and not a program preoccupied with (predictive) planetary astronomy. And it appears that their methods could not, in any discernible way, be adapted to the needs of such a predictive astronomy. Such an agenda would become strongly entrenched with Hipparchus whereby Greek astronomy appears to have assimilated the Babylonian culture of quantitatively predicting planetary positions as a function of time.[32]

In this period, then, a new culture is being formed concerning the construction of (astronomical) theories, which, however, is not unaffected by an already existing culture of observations that itself is being influenced by this new culture. It should be emphasized that the rise of such an enterprise of quantification has not been independent of changes occurring in what was being measured. Starting from plain qualitative observations of phenomena, astronomers proceeded initially to the measurement of apparent diameters, continued to the determination of relative distances and, finally, to gauging planetary motions. Hence, there was first a kind of positional astronomy, in other words, the measurement of (relative) magnitudes and brightness. Then there came the measurements of the distances of the moon, sun, the cosmos and the sky. Aristarchus and Archimedes were fully involved in these. By the time of Hipparchus, however, we have the measurements of orbits and velocities, in ways that are more understandable in terms of dynamics.

PHYSICISTS VERSUS ASTRONOMERS

There are two additional considerations that provide further evidence concerning this new culture. The first is based on a particularly revealing passage of Geminus, the other on the characteristics of the work of the mathematicians who lived in this period and which frame the constitutive features of the new discourse.

Geminus (first century A.D.) in a famous passage of his summary of the *Meteorologica* of Posidonius, quoted by Simplicius through Alexander of Aphrodisias, writes:

> As it [astronomy] is connected with the investigation of quantity, size, and quality of form or shape, it naturally stood in need, in this way, of arithmetic and geometry. The things, then, of which alone astronomy claims to give an account, it is able to establish by means of arithmetic and geometry. Now in many cases the astronomer and the physicist will propose to prove the same point, e.g., that the sun is of great size or that the earth is spherical, but they will not proceed by the same road. The physicist will prove each fact by considerations of essence or substance, of force, of its being better that things should be as they are, or of coming into being and change; the astronomer will prove them by the properties of figures or magnitudes, or by the amount of movement and the time that is appropriate to it. Again, the physicist will in many cases reach the cause by looking to creative force; but the astronomer, when he proves facts from external conditions, is not qualified to judge of the cause, as when, for instance, he declares the earth or the stars to be spherical; sometimes he does not even desire to ascertain the cause, as when he discourses about an eclipse; at other times he invents by way of hypothesis, and states certain expedients by the assumption of which the phenomena will be saved.... For it is no part of the business of an astronomer to know what is by nature suited to a position of rest, and what sort of bodies are apt to move, but he introduces hypotheses under which some bodies remain fixed, while others move, and then considers to which hypotheses the phenomena actually observed in the heaven will correspond.[33]

The importance of this passage for the kinds and the features of the *ideas* that were formulated, has been emphasized by Dreyer.[34] Others have also commented on this passage as characteristic of the different ideas that were being proposed in ancient times. Interestingly, there is a complementary reading of this passage. Geminus is, of course, speaking of ideas, but he is even more insistent about people and their behaviours. Geminus is referring to two kinds of communities, he is telling us about different audiences, and he delineates the two different cultures of these communities. Geminus writes about the different kinds of things people were doing. It is only because the traditional historiography of science was emphasizing almost exclusively the primacy of ideas over the practice of people, that Geminus's unambiguous and straightforward reference to people has been read as a testimonial *only* to their ideas! In fact, Geminus writes about astronomers and physicists, and it is the almost exclusive emphasis on ideas that has distracted from the significance of this passage. And, of course, when we have communities with different constitutive characteristics it is quite justifiable to assume that people test out their views, they propose hypotheses which they then retract, give hints of how they plan to deal with a particular problem, impart information they have about what others of similar dispositions do, and, generally, try to strengthen and enhance those characteristics that give them their individual identity compared to other communities. The new discourse we have been referring to above and the ensuing new practices formed the distinguishing discourse of those whom Geminus refers to as the astronomers. Talking of communities is talking about attitudes, mentalities, disputes and practices. Talking only of ideas, isolated from the practices that they induce to those who either propose or adapt them, is talking, basically, about their logical structure.

Let us now discuss some characteristics of such a mathematical discourse. Aristarchus's *On the sizes and distances of the sun and moon* is the first example in history where we have — in the language of what was later developed — the derivation of trigonometric inequalities. It is claimed that Hipparchus had composed a table of chords and we have no clues that others before him had attempted anything similar, though it may be the case that Archimedes's *Measurement of a circle* is a work indicating that either Archimedes or others may have calculated values of chords. In any case, we witness a 'program' for calculating tables of cords. According to Eutocius, Apollonius in his work under the strange title *Quick-delivery*, had calculated π to a better approximation than the one given by Archimedes, and, thus, we cannot rule out that Apollonius may have drawn up a table of chords. The development of such calculational techniques guarantees results of adequate accuracy. There was, of course, no consensus as to the criteria of when a quantity was indeed accepted as being adequately accurate. Historically the emergence of trigonometry was the outcome of this process which involved the measurement of chords and arcs and the determination of the numerical relationships between sides and angles of triangles.

Another characteristic has been the development of a methodology as well as the technology of astronomical observations. The range of accuracy of these observations affected to some extent the demand for accuracy in the mathematical treatment. Archimedes in the *Sand-reckoner* describes an instrument for measuring the diameter of the sun and we also know that Aristarchus had invented a kind of a sun-dial (σκάφη) the exact details of which we do not have.[35] Furthermore, we know that Hipparchus had extensively used the dioptra, and Ptolemy's spherical astrolabe is most probably due to Hipparchus. Let us also note the great significance of Alexandria during this period, the fact that everyone mentioned in one way or another had visited Alexandria and spent time there, and that the Museum would have had the most sophisticated instrumentation for observations. In addition to all this, astronomers started making critical use of the Babylonian observations. The mathematicians of this period, in addition to their purely mathematical works, also composed a number of texts dealing with practical matters. Some of the better known are Euclid's *Optics* and *Phaenomena*, Ps-Euclid's *Catoptrics,* Archimedes's systematic studies in hydrostatics, and Apollonius's works in astronomy.

It is not, then, unjustified to claim that we are in a period when we discern the establishment and formation of a particular culture for the construction of mathematical theories/models and, hence, a culture that is not unaffected by an already existing yet changing attitude towards observations. The emphasis on quantification and the development of the relevant mathematical methods are intrinsically related to further adjustments, modifications and conformations of the measuring and observational methods. What we are discussing, however, is not so much a matter of new techniques *per se*, but rather of the rise of a new mentality, of a new attitude and, above all, of a new style of thinking and a new discourse. Showing that it is, indeed, possible to calculate what can be measured, and measuring what can be calculated is new mentality. Systematizing and justifying what can be observed is not the same as calculating what can be measured. Geminus's astronomers and physicists may not have had the same aims. But, more importantly, they did not share the same overall culture of how to deal with nature, they had diverging views on how specifically to construct their language, and they developed different styles and mentalities.

CONCLUSIONS

The main argument we have tried to advance in this paper is that the formulation of Aristarchus's heliocentric hypothesis was one of the legitimate alternatives in the attempts to study the cosmos, within a context formed by the new mathematical practice of the early Hellenistic period. This new practice, whose main characteristic was the calculation of quantities by the combined use of geometrical methods with hypotheses whose content was provided from astronomical observations, was not only decisive in the coming-to-be of trigonometry, but it also brought about a culture where the grip of philosophical considerations for the description of the cosmos appears to have been (temporarily) lessened. Interestingly, this is a pattern that we find repeated in other instances in the history of science. There are many cases when the 'rules of the game' are reflected in the hypotheses being formulated, and these hypotheses in their specific form and content need a legitimation which transcends the confines of the theory or model for which they had been formulated in the first place. Aristarchus's heliocentrism could be better understood within a context formed by this new mathematical discourse, which was being formed during the early Hellenistic period and practised by a specific community of (applied) mathematicians. A person adopting such a practice was, in the words of Geminus, "not qualified to judge of the cause — sometimes he does not even desire to ascertain the cause, at other times he invents by way of hypothesis, and states certain expedients by the assumption of which the phenomena will be saved".[36] Failure to bring in these considerations in assessing Aristarchus's heliocentrism, led to accounts by historians of science who express an admiration for how daring Aristarchus's hypothesis was along with their assumption that its formulation was almost certainly accompanied by strong reactions. Apart from the fact that nothing in the extant sources bears witness to such a view, there is no justification whatsoever for transplanting the mentality of the Middle Ages and the Renaissance into the heliocentrism of Antiquity. Of course, most of the objections expressed against heliocentrism on empirical — and perhaps philosophical — grounds during the Middle Ages and the Renaissance, were also raised in Antiquity as well. But it is one thing to be aware of the empirical difficulties related to the heliocentric hypothesis and another to consider them as having the same methodological, ideological and social status in Antiquity as they had in the later centuries after the appropriation of Aristotelianism by Christianity.

The framework of ideas related to the motion of the earth continued to survive for many centuries and in forms involving serious differences from the original Pythagorean views. And these views should not have been considered as bizarre and aberrant views of marginal interest: it appears that their opponents were not dismissive nor were they able simply — and perhaps, easily — to dismiss them. Condescension does not appear to have characterized the attitude of any of those who had adopted the geocentric and geostatic view. Archimedes, it appears, was not dismissive nor did he dismiss Aristarchus's heliocentrism.

The mathematical treatment of astronomical problems was, of course, attempted within a background of a number of astronomical observations of varying accuracy. If one takes into consideration the totality of these observations, then the geocentric and geostatic view does not appear to have indisputable advantages on astronomical grounds over the ideas entertaining some kind of a motion for the earth. By the middle of the third century and independent of when they were originally observed, the following comprised the main astronomical observations known to astronomers: the inequality of the seasons, the phases of the moon, the eclipses of the moon and the sun, the different size of the full moon over the year, the changing luminosity of the planets, the retrogade motion of the planets and their stations, the motion of the sun, the solstices and the equinoxes, and the apparent diameters of the sun and the moon. Had it been the case that astronomical observations furnished evidence for some kind of an advantage of the geocentric and geostatic view, we may be sure that Aristotle would have been the first to emphasise this. Not only do we not find such a pronouncement by Aristotle, we do not even find it in Ptolemy!

If the beginning of the early Hellenistic period provided a rather favourable backdrop to an idiosyncratic pluralism of hypotheses concerning the motion of the earth, by the end of this early Hellenistic period geocentrism appears to have gained an indisputable dominance. We cannot avoid asking the question of what happened, and progressively the geocentric view acquired a privileged status and eventually became the dominant view. One needs to examine this question systematically, but it is possible to speculate, at least, about the main reason for such a development. We feel that the changes in favour of geocentrism must have been related to some mathematical developments that facilitated the re-evaluation of the philosophical considerations in favour of geocentrism. It is known that Apollonius — under certain special conditions which nevertheless were quite general for our purposes — had proved the equivalence of two mathematical models that had been considered as totally different to each other. In one model a body was revolving in a circle around another that was not in the centre, but at a point away from the centre. The motion of the revolving body was regular with respect to the geometrical centre of the circle and not with respect to the body that was situated at the eccentric. In the other model, a body was revolving around an epicycle whose centre was revolving along the circle whose centre was the motionless body. Though we cannot be sure of exactly when the use of eccentrics and epicycles started, we can be quite certain that many individuals must have thought about these issues by the time of Apollonius. Apollonius's proof, and above all Hipparchus's systematic treatment of epicyclic and eccentric motion, eventually became something more than a successful mathematical model. It became a powerful tool that could successfully absorb all kinds of observational data, and hence, slowly, the model itself became an algorithm, inscribing the traces of a new mentality of how to deal with celestial phenomena. Thus the work of these two mathematicians further legitimized the circular and regular motion: *it provided the range of possibilities for the regular circular motion*. It appeared that the methodological directives of both the Academy and the Lyceum could be shown to be a methodology that could actually be visualized and that gave results that were in agreement with the observations. But let us not forget that the methodological directives of both the Academy and the Lyceum were, at the same time, geocentric and geostatic. Perhaps what was metaphysically appealing became physically true.

ACKNOWLEDGEMENTS

We would like to thank Professor G. A. Christodoulou for his helpful remarks concerning the translation of the passage from Archimedes's *Sand-reckoner*. We would also like to thank Rob Iliffe and Michael Hoskin, and the anonymous referees, for their useful comments.

REFERENCES

1. Ptolemy, *Almagest*, translated and annotated by G. J. Toomer (Princeton, 1998), 44–45.
2. T. L. Heath, *Aristarchus of Samos: The ancient Copernicus* (Oxford, 1913), 301.
3. David Fowler talks of arithmetized geometry as follows: “Arithmetized geometry is how we tend to think of geometry today; a line has a length, a number; a rectangle has an area, again a number which is equal to the product of the lengths of its sides; ratios are defined arithmetically, as quotients of numbers; and so on. So the geometry becomes translated into the arithmetical manipulation of numbers ... addition, subtraction, multiplication, division, taking roots, etc. ... and then this arithmetic is later abstracted into algebra.” See D. H. Fowler, “The story of the discovery of incommensurability, revisited”, in *Trends in the historiography of science*, ed. by K. Gavroglu, J. Christianidis and E. Nicolaidis (Dordrecht, 1994), 221–35, p. 229. But he also insists that “early Greek mathematics and astronomy, up to and including Archimedes, was *not* arithmetized” (D. H. Fowler, “Logistic and fractions in early Greek mathematics: A new interpretation”, in *Histoire des fractions, fraction d’histoire*, ed. by P. Benoit, K. Chemla and J. Ritter (Basel, 1992), 133–47, p. 133). Fowler’s view, as expressed in many of his publications, is that the arithmetization of Greek mathematics came later as a result of the introduction of some form of the Babylonian number system in Greek astronomy, after the second century B.C., and that these developments were connected with the works of Hypsicles and Hipparchus. See for example D. H. Fowler, *The mathematics of Plato’s Academy: A new reconstruction* (Oxford, 1990), 222. We shall add some comments to these views later in this paper.
4. See for example T. H. Martin, *Mémoires sur l’histoire des hypothèses astronomiques chez les Grecs et les Romains* (*Mémoires de l’Académie des Inscriptions et des Belles-Lettres*, xxx/2 (1881)); G. Schiaparelli, *Origine del sistema planetario eliocentrico presso i Greci* (*Memorie del R. Instituto Lombardo di Scienze e Lettere: Classe di scienze matematiche e naturali*, xviii (1898)); P. Tannery, *Recherches sur l’histoire de l’astronomie ancienne* (*Mémoires de la Société des Sciences Physiques et Naturelles de Bordeaux*, 4th ser., i (1893)); J. L. E. Dreyer, *History of the planetary systems from Thales to Kepler* (Cambridge, 1906; reprinted as *A history of astronomy from Thales to Kepler* (New York, 1953)); Heath, *op. cit.* (ref. 2); P. Duhem, *Le système du monde: Histoire des doctrines cosmologiques de Platon à Copernique*, i (Paris, 1913); R. v. Erhardt and E. v. Erchardt-Siebold, “Archimedes’ *Sand-reckoner*: Aristarchos and Copernicus”, *Isis*, xxxiii (1942), 578–602; O. Neugebauer, “Archimedes and Aristarchus”, *Isis*, xxxv (1942), 4–6.
5. Plutarch (*Platonicae quaestiones*, viii, 1); Sextus Empiricus (*Adversus mathematicos*, x, 174); Ps-Plutarch (*De placita philosophorum*, ii, 24). The only passage where Aristarchus’s heliocentrism is said to have been considered critically is in Plutarch’s *De facie in orbe lunae*, c. 6.
6. E.g.: “This [the Archimedes account of Aristarchus’s heliocentrism] is stupendous, and would be incredible if we had it from another source” (G. Sarton, *Hellenistic science and culture in the last three centuries B.C.* (New York, 1993), 57); “Mais, dans le courant du IIIe siècle, parut un ouvrage qui *bouleversait* les opinions reçues ...; bien qu’il ne restât pas ignoré même du grand public, le système héliocentrique d’Aristarque n’eut pour ainsi dire aucun succès; seul un astronome du IIe siècle avant J.C. nommé Séleucus ... passe pour l’avoir adopté” (our emphasis) (J. Beaujeu, “Astronomie et géographie mathématique”, in R. Taton (ed.), *Histoire générale des sciences*, i: *La science antique et médiévale des origines à 1450* (Paris, 1994), 356 and 358).
7. Cicero, *Quaestiones academicae priores*, II, 39.
8. B. R. Goldstein and A. C. Bowen, “A new view of early Greek astronomy”, *Isis*, lxxiv (1983), 330–40.
9. Heath’s translation, *op. cit.* (ref. 2), 276.
10. A systematic discussion of the views expressed about this passage can be found in Duhem, *op. cit.*

(ref. 4), 410–18.

11. *Platonicae quaestiones*, viii, 1. See Heath, *op. cit.* (ref. 2), 183.
12. *Ibid*. See Heath, *op. cit.* (ref. 2), 305.
13. *On the heavens*, ii, 13. See Heath, *op. cit.* (ref. 2), 186.
14. See A. Boeckh, *Untersuchungen über das kosmische System des Platon* (Berlin, 1852), 148.
15. See *Quaestiones naturales*, vii, 2.
16. See Goldstein and Bowen, *op. cit.* (ref. 8), 332.
17. Gelon was the son of the king of Syracuse, Hieron II. He co-ruled with his father beginning about 240 B.C. until his death *c*. 216, when he was over fifty years old.
18. W. R. Knorr, "Archimedes and the Elements: Proposal for a revised chronological ordering of the Archimedean corpus", *Archive for history of exact sciences*, x/3 (1978), 211–90.
19. Archimedes's lifetime extended from *c*. 287 to 212 B.C.
20. On Aristarchus's dates see: Heath, *op. cit.* (ref. 2), 299; W. H. Stahl, "Aristarchus of Samos", *Dictionary of scientific biography*, i, 246–50, p. 246; B. E. Wall, "Anatomy of a precursor: The historiography of Aristarchos of Samos", *Studies in history and philosophy of science*, vi (1975), 201–28, pp. 210 seq.
21. Our knowledge of Aristarchus's extant *On the sizes and distances of the sun and moon* is derived from a collection of about twenty manuscripts. These manuscripts include a number of works that are presumed to have made up a unified collection, referred to by Pappus of Alexandria in the sixth book of his *Mathematical collection* under the name of *Treasury of astronomy*. The oldest and best of these manuscripts is from the tenth century and it seems to be the source from which all the other extant manuscripts are derived. See Heath, *op. cit.* (ref. 2), 325. The extant text of the *Sand-reckoner* is also based on a manuscript of the same period. This manuscript (known as Codex A, a name given by Heiberg) had been written in Constantinople in the ninth century; it was then transported to the West and became the archetype of most of the extant works of Archimedes, including the *Sand-reckoner*. This codex disappeared sometime after 1564, but we have various copies of it, made between 1450 and 1550.
22. For the Greek text see J. L. Heiberg, *Archimedes, Opera omnia*, ii (Leipzig, 1913), 218. There have been numerous translations of this passage. See, e.g., Dreyer, *op. cit.* (ref. 4), 136–7; Heath, *op. cit.* (ref. 2), 302; Duhem, *op. cit.* (ref. 4), 420–1; E. J. Dijksterhuis, *Archimedes* (Princeton, 1987), 362–3; Erhardt and Erhardt-Siebold, *op. cit.* (ref. 4), 579. Our translation differs from these in a number of (minor) points. Though these differences have some philological interest, they are not important for the arguments developed in this paper.
23. See ref. 5. Some other evidence about Aristarchus's heliocentrism comes from Plutarch, Aëtius, Sextus Empiricus and Ps-Plutarch. The corresponding passages, which are of minor importance for our purpose, are translated and discussed in Heath's *op. cit.* (ref. 2), 304 seq.
24. R. Netz, *The shaping of deduction in Greek mathematics: A study in cognitive history* (Cambridge, 1999), 284 seq.
25. Erhardt and Erchardt-Siebold, *op. cit.* (ref. 4), 588 seq.
26. The relevant passage is quoted by Heath, *op. cit.* (ref. 2), 221–3.
27. See Goldstein and Bowen, *op. cit.* (ref. 8), 339.
28. Fowler, *The mathematics of Plato's Academy* (ref. 3), 54.
29. G. Huxley, "Aristarchus of Samos and Graeco-Babylonian astronomy", *Greek, Roman and Byzantine studies*, v (1964), 123–31.
30. A. C. Bowen and B. R. Goldstein, "Meton of Athens and astronomy in the late fifth century B.C.", in *A scientific humanist: Studies in memory of Abraham Sachs*, ed. by E. Leichty, M. deJ. Ellis and P. Gerardi (Philadelphia, 1988), 39–81.
31. R. Netz, *op. cit*. (ref. 24), 292 seq.

32. Goldstein and Bowen, *op. cit.* (ref. 8).
33. Heath's translation, *op. cit.* (ref. 2), 275–6.
34. Dreyer, *op. cit.* (ref. 4), 131 seq.
35. Heath, *op. cit.* (ref. 2), 312.
36. Heath, *op. cit.* (ref. 2), 276.